CHAPTER 4

The Integral Forms of the Fundamental Laws

4.1 a) No net force may act on the system: $\underline{\Sigma\vec{F} = 0}$.
b) The energy transferred to or from the system must be zero: $\underline{Q - W = 0}$.
c) If $V_{3n} = \vec{V}_3 \cdot \hat{n}_2 = 10\hat{i} \cdot (-\hat{j}) = \underline{\ 0}$ is the same for all volume elements then

$$\Sigma\vec{F} = \frac{D}{Dt}\vec{V}\int dm, \text{ or } \Sigma\vec{F} = \frac{D}{Dt}(m\vec{V}).$$ Since mass is constant for a system

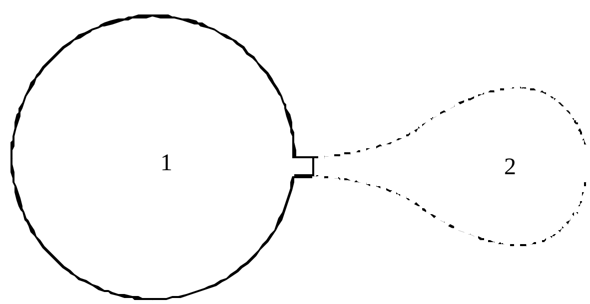
$$\Sigma\vec{F} = m\frac{D\vec{V}}{Dt}.$$ Since $\frac{D\vec{V}}{Dt} = \vec{a}$, $\qquad \Sigma\vec{F} = m\vec{a}.$

4.2 Extensive properties: Mass, m; Momentum, $m\vec{V}$; kinetic energy, $\frac{1}{2}mV^2$;
potential energy, mgh; enthalpy, H.
Associated intensive properties (divide by the mass): unity, 1; velocity, $\vec{V}$; $V^2/2$;
gh; $H/m = h$ (specific enthalpy).
Intensive properties: Temperature, T; time, t; pressure, p; density, *r*; viscosity, *m*.

4.3 (**B**)

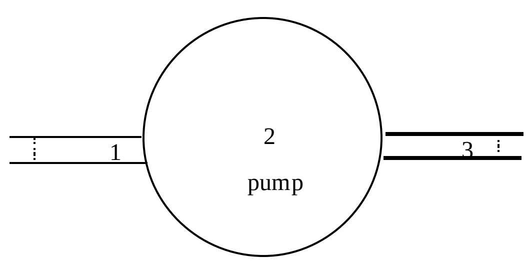
4.4

$$\text{System }(t) = \mathcal{V}_1$$
$$\text{c.v.}(t) = \mathcal{V}_1$$
$$\text{System }(t+\Delta t) = \mathcal{V}_1 + \mathcal{V}_2$$
$$\text{c.v.}(t+\Delta t) = \mathcal{V}_1$$



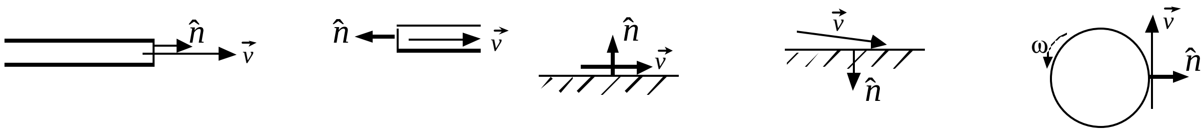
4.5

$$\text{System }(t) = \mathcal{V}_1 + \mathcal{V}_2$$
$$\text{c.v.}(t) = \mathcal{V}_1 + \mathcal{V}_2$$
$$\text{System }(t+\Delta t) = \mathcal{V}_2 + \mathcal{V}_3$$
$$\text{c.v.}(t+\Delta t) = \mathcal{V}_1 + \mathcal{V}_2$$



4.6 a) The energy equation (the 1st law of Thermo).
b) The conservation of mass.
c) Newton's 2nd law.
d) The energy equation.
e) The energy equation.

4.7



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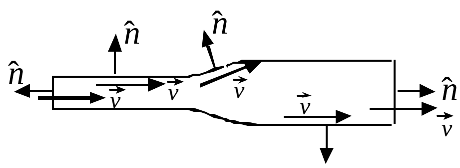
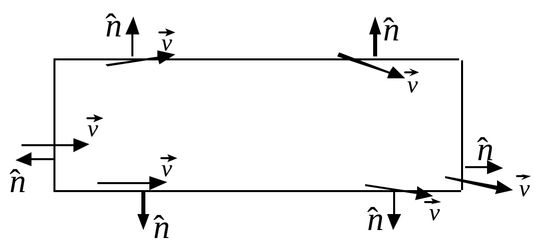


4.9



4.10 $\hat{n}_1 = -\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} = \underline{-0.707(\hat{i}+\hat{j})}$. $\hat{n}_2 = \underline{0.866\hat{i} - 0.5\hat{j}}$. $\hat{n}_3 = \underline{-\hat{j}}$.

$V_{1n} = \vec{V}_1 \cdot \hat{n}_1 = 10\hat{i} \cdot [-0.707(\hat{i}+\hat{j})] = \underline{-7.07 \text{ fps}}$

$V_{2n} = \vec{V}_2 \cdot \hat{n}_2 = 10\hat{i} \cdot (0.866\hat{i} - 0.5\hat{j}) = \underline{8.66 \text{ fps}}$

$V_{3n} = \vec{V}_3 \cdot \hat{n}_2 = 10\hat{i} \cdot (-\hat{j}) = \underline{0}$

4.11 flux = $hr\hat{n} \cdot \vec{V}A$

flux1 = $hr[-0.707(\hat{i}+\hat{j})] \cdot 10\hat{i}A \,/\, 0.707 = \underline{-10hrA}$

flux2 = $hr(0.866\hat{i} - 0.5\hat{j}) \cdot 10\hat{i}A \,/\, 0.866 = \underline{10hrA}$

flux3 = $hr(-\hat{j}) \cdot 10\hat{i}A_3 = \underline{0}$

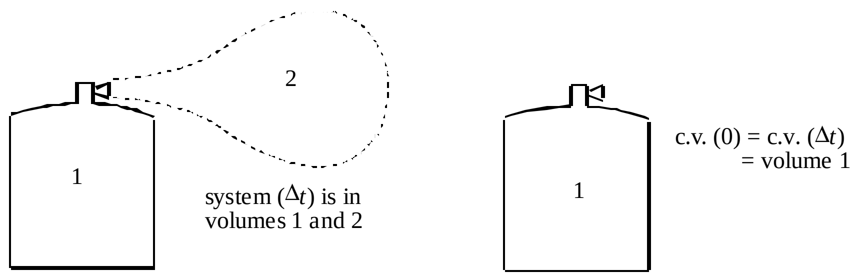
4.12 $(\vec{B}\cdot\hat{n})A \;= 15(0.5\hat{i} + 0.866\hat{j})\cdot\hat{j}(10\times 12)$

$$= 15\times 0.866\times 120 = 1559 \text{ cm}^3$$

Volume = $15 \sin 60^\circ \times 10\times 12 = 1559 \text{ cm}^3$

4.13 The control volume must be independent of time. Since all space coordinates are integrated out on the left, only time remains; thus, we use an ordinary derivative to differentiate a function of time. But, on the right, we note that r and h may be functions of (x, y, z, t); hence, the partial derivative is used.

4.14



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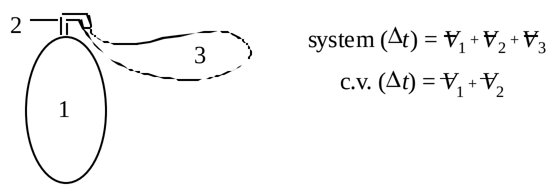

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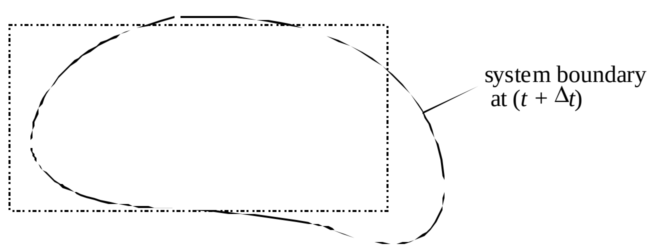


4.17 If fluid crosses the control surface only on areas A_1 and A_2,

$$\int_{c.s.} r\hat{n}\cdot\vec{V}dA = \int_{A_1} r\hat{n}\cdot\vec{V}dA + \int_{A_2} r\hat{n}\cdot\vec{V}dA = 0$$

For uniform flow all quantities are constant over each area:

$$r_1\hat{n}_1\cdot\vec{V}_1\int_{A_1} dA + r_2\hat{n}_2\cdot\vec{V}_2\int_{A_2} dA = 0$$

Let A_1 be the inlet so $\hat{n}_1\cdot\vec{V}_1 = -V_1$ and A_2 be the outlet so $\hat{n}_2\cdot\vec{V}_2 = V_2$. Then

$$-r_1V_1A_1 + r_2V_2A_2 = 0$$

or

$$r_2A_2V_2 = r_1A_1V_1$$

4.18 Use Eq. 4.4.2 with m_V representing the mass in the volume:

$$0 = \frac{dm_V}{dt} + \int_{c.s.} r\hat{n} \cdot \vec{V} dA = \frac{dm_V}{dt} + rA_2V_2 - rA_1V_1$$

$$= \frac{dm_V}{dt} + rQ - \dot{m}.$$

Finally,

$$\underline{\frac{dm_V}{dt} = \dot{m} - rQ.}$$

4.19 Use Eq. 4.4.2 with m_S representing the mass in the sponge:

$$0 = \frac{dm_S}{dt} + \int r\hat{n} \cdot \vec{V} dA = \frac{dm_S}{dt} + rA_2V_2 + rA_3V_3 - rA_1V_1$$

$$= \frac{dm_S}{dt} + \dot{m}_2 + rA_3V_3 - rQ_1.$$

Finally,

$$\underline{\frac{dm_S}{dt} = rQ_1 - \dot{m}_2 - rA_3V_3.}$$

4.20 (**D**) $\dot{m} = rAV = \frac{p}{RT}AV = \frac{200}{0.287 \times 293} p \times 0.04^2 \times 70 = 0.837$ kg/s.

4.21 $A_1V_1 = A_2V_2$. $p \times \frac{1.25^2}{144} \times 60 = p \times \frac{2.5^2}{144} V_2$. $\therefore V_2 = \underline{15 \text{ ft/sec}}$.

$\dot{m} = rAV = 1.94p \frac{1.25^2}{144} \times 60 = \underline{3.968 \text{ slug/sec}}$. $Q = AV = p \frac{1.25^2}{144} \times 60 = \underline{2.045 \text{ ft}^3/\text{ sec}}$.

4.22 $A_1V_1 = A_2V_2$. $p \times .025^2 \times 10 = (2p \times .6 \times .003)V_2$. $\therefore V_2 = \underline{1.736 \text{ m/s}}$.
$\dot{m} = rAV = 1000p \times .025^2 \times 10 = \underline{19.63 \text{ kg/s}}$. $Q = AV = p \times .025^2 \times 10 = \underline{0.01963 \text{ m}^3/\text{ s}}$.

4.23 $\dot{m}_{in} = rA_1V_1 + rA_2V_2$. $200 = 1000\,\pi \times .025^2 \times 25 + 1000\,Q_2$. $\therefore Q_2 = \underline{0.1509 \text{ m}^3/\text{ s}}$.

4.24 $r_1 = \frac{p_1}{RT_1} = \frac{40 \times 144}{1716 \times 520} = .006455 \text{ slug/ft}^3$. $r_2 = \frac{7 \times 144}{1716 \times 610} = .000963 \text{ slug/ft}^3$.

$\dot{m} = rAV$. $\therefore V_1 = \frac{\dot{m}}{r_1A_1} = \frac{.2}{(p \times 2^2 / 144).006455}$. $\therefore V_1 = \underline{355 \text{ fps}}$.

$\dot{m}_2 = 0.2 = .000963 \times (2 \times 3/144)V_2$. $\therefore V_2 = \underline{4984 \text{ fps}}$.

4.25 $r_1 A_1 V_1 = r_2 A_2 V_2$. $r_1 = \frac{p_1}{RT} = \frac{500}{.287\times 393} = 4.433\frac{\text{kg}}{\text{m}^3}$. $r_2 = \frac{1246}{.287\times 522} = 8.317\frac{\text{kg}}{\text{m}^3}$

$4.433\ p \times .05^2 \times 600 = 8.317\ p \times .05^2\ V_2$. $\therefore V_2 = \underline{319.8\text{ m/s}}$.

$\dot{m} = r_1 A_1 V_1 = \underline{20.89\text{ kg/s}}$. $Q_1 = A_1 V_1 = \underline{4.712\text{ m}^3/\text{ s}}$. $Q_2 = \underline{2.512\text{ m}^3/\text{ s}}$.

4.26 $r_1 A_1 V_1 = r_2 A_2 V_2$

$$\frac{p_1}{RT_1} A_1 V_1 = \frac{p_2}{RT_2} A_2 V_2$$

$$\frac{200}{293} p \times 0.05^2 \times 40 = \frac{120}{T_2} p \times 0.03^2 \times 120.$$

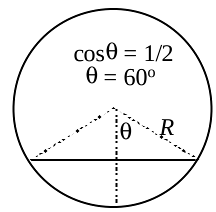
$\therefore T_2 = 189.9$ K or $\underline{-83^\circ\text{C}}$.

4.27 a) $A_1 V_1 = A_2 V_2$. $(2\times 1.5 + 1.5\times 1.5)\ 3 = p\frac{d_2^2}{4}\times 2$. $\therefore d_2 = \underline{3.167\text{ m}}$

b) $(2\times 1.5 + 1.5\times 1.5)\ 3 = p\frac{d_2^2}{4}\times\frac{2}{2}$. $\therefore d_2 = \underline{4.478\text{ m}}$

c) $(2\times 1.5 + 1.5\times 1.5)\ 3 = \left(\frac{1}{3}pR^2 - \frac{R}{2}\times .866R\right)\times 2$.

$\therefore R = 3.581$ m. $\therefore d_2 = \underline{7.162\text{ m}}$



4.28 (**A**) Refer to the circle of Problem 4.27:

$$Q = AV = (p\times 0.4^2 \times\frac{75.7\times 2}{360} - 0.10\times 0.40\times\sin 75.5^\circ)\times 3 = 0.516\text{ m}^3\text{/s}.$$

4.29 a) $v = 10\left(1-\frac{r}{r_0}\right)$. $pr_0^2 V = \int_0^{r_0} v\,dA = \int_0^{r_0} 10\left(1-\frac{r}{r_0}\right)2prdr = 20p\int_0^{r_0}\left(r-\frac{r^2}{r_0}\right)dr$.

$$\therefore V = \frac{20}{r_0^2}\left(\frac{r_0^2}{2}-\frac{r_0^2}{3}\right) = \frac{10}{3} = \underline{3.333\text{ m/s}}.$$

$\dot{m} = rAV = 1000\times p\times .04^2\times 3.33 = \underline{16.75\text{kg/s.}}$ $Q = AV = \underline{0.01675\text{ m}^3/\text{ s}}$.

b) $v = 10\left(1-\frac{r^2}{r_0^2}\right)$. $pr_0^2 V = \int_0^{r_0} 10\left(1-\frac{r^2}{r_0^2}\right)2prdr = 20p\left(\frac{r_0^2}{2}-\frac{r_0^2}{4}\right)$. $\therefore V = \underline{5\text{ m/s}}$

$\dot{m} = rAV = 1000\times p\times .04^2\times 5 = \underline{25.13\text{ kg/ s.}}$ $Q = AV = \underline{0.02513\text{ m}^3/\text{ s.}}$

c) $v = 20\left(1-\frac{r}{r_0}\right)$. $pr_0^2 V = \int_{r_0/2}^{r_0} 20\left(1-\frac{r}{r_0}\right)2prdr + 10pr_0^2/4$. $\therefore V = \underline{5.833\text{ m/s}}$

$\dot{m} = rAV = 1000\times p\times .04^2\times 5.833 = \underline{29.32\text{ kg/ s.}}$ $Q = \underline{0.02932\text{ m}^3/\text{ s}}$.

4.30 a) Since the area is rectangular, $V = \underline{5\text{ m/s}}$.

$$\dot{m} = rAV = 1000 \times .08 \times .8 \times 5 = \underline{320\text{ kg/ s.}} \qquad Q = \frac{\dot{m}}{r} = \underline{0.32\text{ m}^3/\text{ s}}.$$

b) $v = 40\left(\frac{y}{h} - \frac{y^2}{h^2}\right)$ with $y = 0$ at the lower wall.

$$\therefore Vhw = \int_0^h 40\left(\frac{y}{h} - \frac{y^2}{h^2}\right)wdy = 40 \times \frac{h}{6}w. \quad \therefore V = \underline{6.667\text{ m/s}}.$$

$$\dot{m} = rAV = 1000 \times .08 \times .8 \times 6.667 = \underline{426.7\text{ kg/ s.}} \qquad Q = \underline{0.4267\text{ m}^3/\text{ s}}.$$

c) $V \times .08 = 10 \times .04 + 5 \times .02 + 5 \times .02. \quad \therefore V = \underline{7.5\text{ m/s}}.$

$$\dot{m} = rAV = 1000 \times .08 \times .8 \times 7.5 = \underline{480\text{ kg/ s.}} \qquad \dot{Q} = \frac{\dot{m}}{r} = \underline{0.48\text{ m}^3/\text{ s}}.$$

4.31 a) $A_1V_1 = \int v_2 dA. \qquad p \times \left(\frac{1}{24}\right)^2 \times 6 = \int_0^{r_0} v_{max}\left(1 - \frac{r^2}{r_0^2}\right)2prdr = 2pv_{max}\frac{r_0^2}{4}.$

With $r_0 = \frac{1}{24}, \qquad v_{max} = 12\text{ fps}. \qquad \therefore v(r) = \underline{12(1 - 576r^2)\text{ fps}}.$

b) $A_1V_1 = \int v_2 dA. \qquad \frac{1}{12} \times w \times 6 = \int_{-h}^{h} v_{max}\left(1 - \frac{y^2}{h^2}\right)wdy = v_{max}w\frac{4h}{3}.$

With $h = \frac{1}{24}, \qquad v_{max} = 9\text{ fps}. \qquad \therefore v(y) = \underline{9(1 - 576y^2)\text{ fps}}.$

c) $A_1V_1 = \int v_2 dA. \qquad p \times 0.01^2 \times 2 = \int_0^{r_0} v_{max}\left(1 - \frac{r^2}{r_0^2}\right)2p\,rdr = 2p\,v_{max}\frac{r_0^2}{4}.$

With $r_0 = 0.01$ m, $\qquad v_{max} = 4\text{ m/s}. \qquad \therefore v(r) = \underline{4(1 - 10000r^2)\text{ m/s}}.$

d) $\hat{n} \qquad 0.02 \times w \times 2 = \int_{-h}^{h} v_{max}\left(1 - \frac{y^2}{h^2}\right)wdy = v_{max}w\frac{4h}{3}.$

With $h = 0.01$ m, $\qquad v_{max} = 3\text{ m/s}. \qquad \therefore v(y) = \underline{3(1 - 10000y^2)\text{ m/s}}.$

4.32 If $dm/dt = 0$, then $r_1A_1V_1 = r_2A_2V_2 + r_3A_3V_3$. In terms of $\dot{m}_2$ and Q_3 this becomes, letting $r_1 = r_2 = r_3$,

$$1000 \times p \times 0.02^2 \times 12 = \dot{m}_2 + 1000 \times 0.01. \qquad \therefore \dot{m}_2 = \underline{5.08\text{ kg/ s}}.$$

4.33 $\int_0^{r_1} v_1 dA = A_2V_2. \qquad \int_0^{r_1} v_{max}\left(1 - \frac{r^2}{r_1^2}\right)2prdr = p \times .0025^2 \times 2.$

$$\therefore 2pv_{max}\frac{.005^2}{4} = p \times .0025^2 \times 2. \qquad \therefore v_{max} = 1\text{ m/s}. \qquad \therefore v(r) = \underline{\left(1 - \frac{r^2}{.005^2}\right)\text{m/ s}}.$$

4.34 $\dot{m}_{in} = \dot{m}_{out} + \dot{m}. \quad r\times.2\times 2\times 10 = \left[r\int_0^{.1} 10(20y - 100y^2)2dy + r\times.1\times 2\times 10 \right] + \dot{m}.$

Note: We see that at y = 0.1 m the velocity $u(.1)$ = 10 m / s. Thus we integrate to y = 0.1, and between y = 0.1 and 0.2 the velocity u = 10.

$4r = \left[\frac{4}{3} r + 2r \right] + \dot{m}. \qquad \therefore \dot{m} = 0.6667\,r$ = <u>0.82 kg / s</u>.

4.35 $V_1 h_1 = \int_0^h u(y)dy. \qquad 10\times.05 = \int_0^h 10(20y - 100y^2)dy$

$$= 10\left(10h^2 - \frac{100}{3}h^3 \right).$$

$\therefore 666.7\,h^3 - 200\,h^2 = -1$. This can be solved by trial-and-error:

$h = .06$: $\quad -.576 \overset{?}{=} -1. \qquad h = .07$: $\quad -.751 \overset{?}{=} -1.$

$h = .08$: $\quad -.939 \overset{?}{=} -1. \qquad h = .083$: $\quad -.997 \overset{?}{=} -1.$

$h = .084$: $\quad -1.016 \overset{?}{=} -1. \qquad \therefore h = 0.0832$: or <u>8.32 cm</u>.

Note: Fluid does not cross a streamline so all the flow that enters on the left leaves on the right. The streamline simply moves further from the wall.

4.36 $\dot{m} = \int rVdA = \int_0^{1/3} 2.2\left(1 - .3545y\right)(6y - 9y^2)2\times 5dy$

$$= 22\int_0^{1/3}\left(6y - 2.127y^2 - 9y^2 + 3.19y^3\right)dy$$ = <u>4.528 slug / sec</u>.

$\overline{V} = \frac{2}{3}u_{\max} = \frac{2}{3}\times 2 = \frac{4}{3}$ fps. (See Prob. 4.31b).

$\overline{r} = \frac{2.2 + 1.94}{2}$ = 2.07 slug / ft³. $\qquad \therefore \overline{r}\overline{V}A = 2.07\times\frac{4}{3}\times\left(5\times\frac{1}{3}\right)$ = <u>4.6 slug / sec</u>.

Thus, $\overline{r}\overline{V}A \neq \dot{m}$ since $r = r(y)$ and $V = V(y)$ so that $\overline{r}\overline{V} \neq \overline{rV}$.

4.37 $A_1V_1 = A_2V_2. \qquad p\times.01^2\times 8 = (2p\times.2\times.04)V_2\cos 30^\circ. \qquad \therefore V_2$ = <u>0.05774 m / s</u>.

4.38 $2000\times\frac{4}{3}p\times.0015^3\,\frac{\text{m}^3\text{ of H}_2\text{O}}{\text{m}^3\text{ of air}}\times 9000\times 5\,\frac{\text{m}^3\text{ of air}}{\text{s}} = 1.5\times(1.5h). \qquad \therefore h$ = <u>0.565 m</u>.

4.39 Use Eq. 4.3.3: $\qquad 0 = \int\frac{\P r}{\P t}d V + r_1\vec{V}_1\cdot\hat{n}_1A_1. \qquad \vec{V}_1\cdot\hat{n}_1 = -V_1.$

$\therefore r_1A_1V_1 = \frac{\P r}{\P t}V_{\text{tire}}. \qquad \frac{(37 + 14.7)144}{1716\times 520}\times p\times\left(\frac{1}{96}\right)^2\times 180 = \frac{\P r}{\P t}\times 17.$

$$\therefore \frac{\P r}{\P t} = \underline{3.01\times10^{-5}\ \frac{\text{slug}}{\text{ft}^3-\text{sec}}}.$$

4.40 $\dot{m}_{\text{in}} = \dot{m}_2 + \dot{m}_3$. $\overline{V}_1 = 20$ m/s (see Prob. 4.31c).

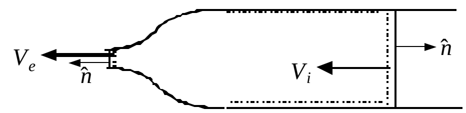
$20\times1000p\times.02^2 = 10 + 1000p\times.02^2\,\overline{V}_3$. $\therefore \overline{V}_3 = \underline{12.04\text{ m/s}}$.

4.41 $0 = \frac{d}{dt}m_{c.v.} + \dot{m}_{net} = \frac{d}{dt}m_{c.v.} + \dot{m}_2 + \dot{m}_3 - \dot{m}_1$

$\therefore \frac{d}{dt}m_{c.v.} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 = 1000\times p\times.02^2\times20 - 10 - 1000p\times.02^2\times10$

$= \underline{2.57\text{ kg/s.}}$

4.42 The control surface is close to the interface at the instant shown.



$\therefore V_i$ = interface velocity.

$r_e A_e V_e = r_i A_i V_i$.

$1.5\times p\times.15^2\times300 = \frac{8000}{.287\times673}p\times1^2 V_i$.

$\therefore V_i = \underline{0.244\text{ m/s.}}$

4.43 Assume an incompressible flow:

$4Q_1 = A_2V_2$. $4\times1500/\ 60 = (2\times4)V_2$. $\therefore V_2 = \underline{12.5\text{ fps}}$.

4.44 For an incompressible flow (low speed air flow)

$\int_{A_1} u\,dA = A_2V_2$. $\int_0^{0.2} 20y^{1/5}\times0.8dy = p\times0.15^2V_2$.

$20\times0.8\frac{5}{6}0.2^{6/5} = p\times0.15^2V_2$. $\therefore V_2 = \underline{27.3\text{ m/ s}}$.

4.45 $A_1V_1 + \int v_2 dA = A_eV_e$

$p(0.1^2 - 0.025^2)\times4 + \int_0^{0.025} 200\left(1 - \frac{r^2}{0.025^2}\right)2prdr = p\times0.1^2V_e$

$0.1178 + 0.1963 = 0.0314V_e$. $\therefore V_e = \underline{10.0\text{ m/ s}}$.

4.46 Draw a control volume around the entire set-up:

$0 = \frac{dm_{tissue}}{dt} + rV_2A_2 - rV_1A_1$

$= \dot{m}_{tissue} + rp\left(\frac{d_2^2 - d^2}{4}\right)\dot{h}_2 - rp(h_1\tan f)^2\dot{h}_1$

or

$$\dot{m}_{tissue} = \underline{\rho\pi\left[\frac{d^2 - d_2^2}{4}\dot{h}_2 + h_1^2\dot{h}_1 \tan^2\phi\right]}.$$

4.47 The width w of the channel is constant throughout the flow. Then

$$0 = \frac{dm}{dt} + \rho A_2 V_2 - \rho A_1 V_1. \qquad 0 = \frac{d}{dt}(\rho whL) + \rho A_2 V_2 - \rho A_1 V_1$$

$$0 = \rho\frac{dh}{dt}w \times 100 + \rho 0.2w \times 8 - \rho 4w \times 0.2. \qquad \therefore \dot{h} = \underline{0.008 \text{ m/s}}.$$

4.48 $0 = \frac{dm}{dt} + \rho A_2 V_2 - \rho A_1 V_1$

$$= \dot{m} + 1000(\pi \times 0.003^2 \times 0.02 - 10 \times 10^{-6}/60). \qquad \therefore \dot{m} = \underline{3.99 \times 10^{-4} \text{ kg/s}}.$$

4.49 $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$. $\quad \dot{m}_1 = \rho_2 A_2 V_2$.

$$400e^{-10/100} \times 10^{-6} \times 900 = 0.2 \times \pi \times 0.05^2 V_e. \qquad \therefore V_e = \underline{207 \text{ m/s}}.$$

4.50 $0 = \frac{dm}{dt} + \rho_3 Q_3 - \rho_1 A_1 V_1 - \dot{m}_2$ where $m = \rho Ah$.

a) $0 = 1000\pi \times 0.6^2 \dot{h} + 1000 \times 0.6/60 - 1000\pi \times 0.02^2 \times 10 - 10.$

$\therefore \dot{h} = 0.0111 \text{ m/s}$ or $\underline{11.1 \text{ mm/s}}$

b) $0 = 1000\pi \times 0.6^2 \dot{h} + 1000 \times 0.01 - 0 - 20.$

$\therefore \dot{h} = 0.00884 \text{ m/s}$ or $\underline{8.84 \text{ mm/s}}$.

c) $0 = 1000\pi \times 0.6^2 \dot{h} + 1000 \times 1.0/60 - 1000\pi \times 0.02^2 \times 5 - 10.$

$\therefore \dot{h} = 0.000339 \text{ m/s}$ or $\underline{0.339 \text{ mm/s}}$.

4.51 $A_1 V_1 = A_2 V_2$ where A_2 is an area just under the top surface.

a) $\pi \times 0.02^2 \times 10e^{-t/10} = \pi \times (h \tan 60^\circ)^2 \frac{dh}{dt}$

$\therefore h^2 dh = 0.001333e^{-t/10}dt. \qquad \therefore h^3 = -0.04e^{-t/10} + 0.04.$

Finally,

$h(t) = \underline{0.342(1 - e^{-t/10})^{1/3}}$.

b) $0.04 \times 10 \times 10e^{-t/10} = (h \tan 60^\circ) \times 10\dot{h}$

$\therefore hdh = 0.2309e^{-t/10}dt. \qquad \therefore h^2 = -4.62e^{-t/10} + 4.62.$

Finally,

$h(t) = \underline{2.15(1 - e^{-t/10})^{1/2}}$.

4.52 $\dot{W} = Tw + pAV + m\frac{du}{dy}A_{belt}$

$$= 20\times 500\times 2p/\ 60 + 400\times 0.4\times 0.5\times 10 + 1.81\times 10^{-5}\times 100\times 0.5\times 0.8$$
$$= 1047 + 800 + 0.000724 = \underline{1847\ \text{W}}$$

4.53 If the temperature is essentially constant, the internal energy of the c.v. does not change and the flux of internal energy into the pipe is the same as that leaving the pipe. Hence, the two integral terms are zero. The losses are equal to the heat transfer exiting the pipe.

4.54 80% of the power is used to increase the pressure while 20% increases the internal energy ($\dot{Q} = 0$ because of the insulation). Hence,

$$\dot{m}\Delta\tilde{u} = 0.2\dot{W}$$
$$1000\times 0.02\times 4.18\Delta T = 0.2\times 500. \qquad \therefore \Delta T = \underline{0.836\ ^\circ\text{C}}.$$

4.55 (**D**) $\frac{\dot{W}_P}{gQ} = \frac{V_2^2 - V_1^2}{2g} + \frac{p_2 - p_1}{g}$. $\qquad \frac{\dot{W}_P}{g\times 0.040} = \frac{1200-200}{g}$.

$$\therefore \dot{W}_P = 40\ \text{kW} \qquad \text{and energy req'd} = \frac{40}{0.85} = 47.1\ \text{kW}.$$

4.56 $\dot{W}_P = \frac{QgH_P}{h_p}$. $\qquad 5\times 746 = \frac{Q\times 9800\times 20}{0.87}$. $\qquad \therefore Q = \underline{0.01656\ \text{m}^3/\text{s}}$.

4.57 $-\frac{\dot{W}_T}{\dot{m}g} = -40\times 0.89.$

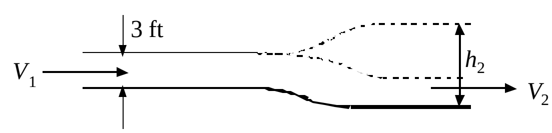
a) $\dot{W}_T = 40\times 0.89\times 200\times 9.81 = \underline{69\ 850\ \text{W}}$

b) $\dot{W}_T = 40\times 0.89\times(90\ 000/\ 60)\times 9.81 = \underline{523\ 900\ \text{W}}$

c) $\dot{W}_T = 40\times 0.89\times(8\times 10^6\ /\ 3600)\times 9.81 = \underline{776\ 100\ \text{W}}$

4.58 $-\frac{\dot{W}_T}{r\,AVg} = h_T\Delta z.$ $\qquad \frac{10000000}{100\times 3\times 60\times V\times 9.8} = 0.89\times 50.$ $\qquad \therefore V = \underline{1.273\ \text{m/s}}$

4.59 $\frac{V_1^2}{2g} + \frac{p_1}{g} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{g} + z_2.$



$$\frac{12^2}{2\times 32.2} + 6 = \frac{36^2}{64.4h_2^2} + h_2.$$

$$8.236 = \frac{20.1}{h_2^2} + h_2.$$

Continuity: $3\times 12 = h_2 V_2$.

This can be solved by trial-and-error.

$h_2 = 8'$: $8.24 \overset{?}{=} 8.31$ $h_2 = 7.9'$: $8.24 \overset{?}{=} 8.22$ $\because h_2 = \underline{7.93'}$.

$h_2 = 1.8'$: $8.24 \overset{?}{=} 8.00$ $h_2 = 1.75'$: $8.24 \overset{?}{=} 8.31$ $\because h_2 = \underline{1.76'}$.

4.60 $\frac{V_1^2}{2g} + z_1 = \frac{V_2^2}{2g} + z_2 + h_L$. $\therefore \frac{4^2}{2\times 9.81} + 2 = \frac{16}{2\times 9.81 h_2^2} + h_2 + 0.2.$

$\therefore 2.615 = 0.815/\ h_2^2 + h_2$. Trial-and-error provides the following:

$h_2 = 2.5$: $2.615 \overset{?}{=} 2.63$ $h_2 = 2.45$: $2.615 \overset{?}{=} 2.59$. $\therefore h_2 = \underline{2.47 \text{ m}}$

$h_2 = 0.65$: $2.615 \overset{?}{=} 2.58$ $h_2 = 0.64$: $2.615 \overset{?}{=} 2.63$. $\therefore h_2 = \underline{0.646 \text{ m}}$

4.61 Manometer: Position the datum at the top of the right mercury level.

$$9810\times.4 + 9810 z_2 + p_2 + \frac{V_2^2}{2}\times 1000 = (9810\times 13.6)\times.4 + 9810\times 2 + p_1$$

Divide by $g = 9810$: $.4 + z_2 + \frac{p_2}{g} + \frac{V_2^2}{2g} = 13.6\times.4 + 2 + \frac{p_1}{g}$. (1)

Energy: $\frac{V_1^2}{2g} + \frac{p_1}{g} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{g} + z_2$. (2)

Subtract (1) from (2): With $z_1 = 2$ m, $\frac{V_1^2}{2g} = 12.6\times.4$. $\therefore V_1 = \underline{9.94 \text{ m/s}}$

4.62 The manometer equation (see Prob. 4.61) is

$$0.4 + z_2 + \frac{p_2}{g} + \frac{V_2^2}{2g} = 13.6\times.4 + 2 + \frac{p_1}{g}. \qquad (1)$$

Energy: $\frac{V_1^2}{2g} + \frac{p_1}{g} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{g} + z_2 + 0.05\frac{V_2^2}{2g}$. (2)

Subtract (1) from (2): With $z_1 = 2$ m, and with $V_2 = 4V_1$ (continuity)

$\frac{1.8 V_1^2}{2g} = 12.6\times 0.4$. $\therefore V_1 = \underline{7.41 \text{ m/s}}$.

4.63 (**A**) $0 = \frac{\cancel{V_2^2} - V_1^2}{2g} + \frac{p_2 - \cancel{p_1}}{g}$. $0 = \frac{-120^2}{2\times 9.8} + \frac{p_2}{9810}$. $\therefore p_2 = 7\ 200000$ Pa.

4.64 $Q = 120\times 0.002228 = \pi\times\left(\frac{1}{12}\right)^2 V_1$. $\therefore V_1 = 12.25$ fps.

Continuity: $p \times \left(\frac{1}{12}\right)^2 V_1 = p \times \left(\frac{1.5}{12}\right)^2 V_2$. $\therefore V_2 = 5.44$ fps.

Energy: $\frac{V_1^2}{2g} + \frac{p_1}{g} = \frac{V_2^2}{2g} + \frac{p_2}{g} + 0.37\frac{V_1^2}{2g}$.

$\therefore p_2 = 60 \times 144 + 62.4\left[0.63\frac{12.25^2}{64.4} - \frac{5.44^2}{64.4}\right]$ = 8702.9 psf or <u>60.44 psi</u>

4.65 $Q = 600 \times 10^{-3} / 60 = \pi \times .02^2\ V_1$. $\therefore V_1 = 7.958$ m / s.

$a = \frac{1}{A\bar{V}^3}\int V^3 dA = \frac{1}{0.02 \times w \times 6.67^3}\int_0^{0.02} 10^3\left(1 - \frac{y^2}{0.02^2}\right)^3 wdy$

$V_2 = \frac{A_1 V_1}{A_2} = \frac{.04^2 \times 7.958}{.06^2}$ = 3.537 m / s.

Energy: $\frac{V_1^2}{2g} + \frac{p_1}{g} = \frac{V_2^2}{2g} + \frac{p_2}{g} + h_L$.

$\therefore h_L = \frac{7.958^2 - 3.537^2}{2 \times 9.81} + \frac{690\ 000 - 700\ 000}{9810}$ = <u>1.571 m</u>

4.66 $V_1 = Q / A_1 = \frac{0.08}{p \times .03^2}$ = 28.29 m / s. $\therefore V_2 = 9V_1 = 254.6$ m / s.

Energy: $\frac{V_1^2}{2g} + \frac{p_1}{g} = \frac{V_2^2}{2g} + \frac{p_2}{g} + .2\frac{V_1^2}{2g}$.

$\therefore p_1 = 9810\left[\frac{254.6^2}{2 \times 9.81} - 0.8\frac{28.29^2}{2 \times 9.81}\right]$ = <u>32.1×10^6 Pa</u>.

4.67 a) Across the nozzle: $p \times .07^2 V_1 = p \times .025^2 V_2$. $\therefore V_2 = 7.84\ V_1$.

Energy: $\frac{V_1^2}{2g} + \frac{p_1}{g} = \frac{V_2^2}{2g} + \frac{p_2}{g}$. $\therefore p_1 = 9810\frac{7.84^2 - 1}{2 \times 9.81} V_1^2$.

For the contraction: $p \times .07^2 V_1 = p \times .05^2 V_3$. $\therefore V_3 = 1.96\ V_1$.

Energy: $\frac{V_1^2}{2g} + \frac{p_1}{g} = \frac{V_3^2}{2g} + \frac{p_3}{g}$.

Manometer: $g \times .15 + p_1 = 13.6g \times .15 + p_3$. $\therefore \frac{p_1}{g} = 12.6 \times .15 + \frac{p_3}{g}$.

Subtract the above 2 eqns: $\frac{V_1^2}{2g} + 12.6 \times .15 = \frac{V_3^2}{2g} = 1.96^2\frac{V_1^2}{2g}$.

$\therefore (1.96^2 - 1)V_1^2 = 12.6 \times .15 \times 2g$. $\therefore V_1 = 3.612$ m / s. $\therefore p_1$ = <u>394 400 Pa</u>.

From the reservoir surface to section 1:

$$\frac{V_0^2}{2g}+\frac{p_0}{g}+z_0=\frac{V_1^2}{2g}+\frac{p_1}{g}+z_1$$

$$H=\frac{3.612^2}{19.62}+\frac{394\ 400}{9810}=\underline{40.0\text{ m}}.$$

b) Manometer: $g\times.2+p_1=13.6g\times.2+p_3.$ $\therefore\frac{p_1}{g}=12.6\times.2+\frac{p_3}{g}.$

Energy: $\frac{V_1^2}{2g}+\frac{p_1}{g}=\frac{V_3^2}{2g}+\frac{p_3}{g}.$ Also, $V_3 = 1.96\ V_1$.

$\therefore\frac{V_1^2}{2g}+12.6\times.2=\frac{1.96^2V_1^2}{2g}.$ $\therefore V_1 = 4.171$ m/s.

The nozzle is the same as in part (a): $\therefore p_1 = \underline{534\ 700\text{ Pa}}$.

From the reservoir surface to the nozzle exit:

$\frac{V_0^2}{2g}+\frac{p_0}{g}+z_0=\frac{V_2^2}{2g}+\frac{p_2}{g}+z_2.$ $\therefore H=\frac{V_2^2}{2g}=\frac{32.7^2}{2\times9.81}=\underline{54.5\text{ m}}.$

4.68 a) Energy: $\frac{V_0^2}{2g}+\frac{p_0}{g}+z_0=\frac{V_2^2}{2g}+\frac{p_2}{g}+z_2.$ $\therefore V_2=\sqrt{2gz_0}=\sqrt{2\times9.81\times2.4}=6.862$ m/s.

$Q = AV = .8\times1\times6.862 = \underline{5.49\text{ m}^3/\text{s}}.$

For the second geometry the pressure on the surface is zero but it increases with depth. The elevation of the surface is 0.8 m.

$\therefore z_0=\frac{V_2^2}{2g}+h.$ $\therefore V_2=\sqrt{2g(z_0-h)}=\sqrt{2\times9.81\times2}=6.264$ m/s.

$\therefore Q = .8\times6.264 = \underline{5.01\text{ m}^3/\text{s}}.$

Note: z_0 is measured from the channel bottom in the 2nd geometry.

$\therefore z_0 = H + h.$

b) $\frac{V_0^2}{2g}+\frac{p_0}{g}+z_0=\frac{V_2^2}{2g}+\frac{p_2}{g}+z_2.$ $\therefore V_2=\sqrt{2gz_0}=\sqrt{2\times32.2\times\left(6+\frac{2}{2}\right)}=21.23$ fps.

$\therefore Q = AV = (2\times1)\times21.23 = \underline{42.5\text{ cfs}}.$

For the second geometry, the bottom is used as the datum:

$\therefore z_0=\frac{V_2^2}{2g}+0+h.$ $\therefore\frac{V_2^2}{2g}=(H+h)-h.$

$\therefore V_2=\sqrt{2gH}=\sqrt{2\times32.2\times6}=19.66$ fps. $\therefore Q = \underline{39.3\text{ cfs}}.$

4.69 From the reservoir surface to the exit: Continuity:

$$\frac{V_0^2}{2g}+\frac{p_0}{g}+z_0=\frac{V_2^2}{2g}+\frac{p_2}{g}+z_2+K\frac{V_1^2}{2g}. \qquad V_1=V_2\frac{.03^2}{.08^2}=.1406\ V_2.$$

$$10=\frac{V_2^2}{2g}+5\times\frac{.1406^2V_2^2}{2g}$$

$\therefore V_2 = 13.36$ m / s. $\qquad \therefore Q = 13.36\times\pi\times .015^2 = \underline{0.00944 \text{ m}^3/\text{ s}}$.

The velocity in the pipe is $V_1 = 1.878$ m / s.

Energy $0\rightarrow A$: $10=\frac{1.878^2}{2\times 9.81}+\frac{p_A}{9810}+.8\frac{1.878^2}{2\times 9.81}+3.$ $\qquad \therefore p_A = \underline{65\ 500 \text{ Pa}}$.

Energy $0\rightarrow B$: $10=\frac{1.878^2}{2\times 9.81}+\frac{p_B}{9810}+2.0\frac{1.878^2}{2\times 9.81}+10.$ $\qquad \therefore p_B = \underline{-5290 \text{ Pa}}$.

Energy $0\rightarrow C$: $10=\frac{1.878^2}{2\times 9.81}+\frac{p_C}{9810}+12+2.8\frac{1.878^2}{2\times 9.81}.$ $\qquad \therefore p_C = \underline{-26\ 300 \text{ Pa}}$.

Energy $0\rightarrow D$: $10=\frac{1.878^2}{2\times 9.81}+\frac{p_D}{9810}+0+5\frac{1.878^2}{2\times 9.81}.$ $\qquad \therefore p_D = \underline{87\ 500 \text{ Pa}}$.

4.70 $\frac{V_0^2}{2g}+\frac{p_0}{g}+z_0=\frac{V_2^2}{2g}+\frac{p_2}{g}+z_2.$ $\qquad \frac{80\ 000}{9810}+4=\frac{V_2^2}{2\times 9.81}.$ $\qquad \therefore V_2 = 19.04$ m / s.

a) $Q=A_2V_2=p\times .025^2\times 19.04 = \underline{0.0374 \text{ m}^3/\text{ s}}$.

b) $Q=A_2V_2=p\times .09^2\times 19.04 = \underline{0.485 \text{ m}^3/\text{ s}}$.

c) $Q=A_2V_2=p\times .05^2\times 19.04 = \underline{0.1495 \text{ m}^3/\text{ s}}$.

4.71 a) $\frac{p_0}{g}+z_0=\frac{V_2^2}{2g}+1.54\frac{V_1^2}{2g}.$ $\qquad \frac{80\ 000}{9810}+4=\frac{16V_1^2}{2g}+1.54\frac{V_1^2}{2g}.$ $\qquad \therefore V_1 = 3.687$ m / s.

$Q=A_1V_1=p\times .05^2\times 3.687 = \underline{0.0290 \text{ m}^3/\text{ s}}$.

b) $A_1V_1=A_2V_2.$ $\qquad V_1=\frac{.09^2}{.05^2}V_2=3.24V_2.$

$\frac{80\ 000}{9810}+4=\frac{V_2^2}{2g}+2.3\frac{3.24^2V_2^2}{2g}.$ $\quad \therefore V_2 = 3.08$ m / s. $\quad \therefore Q=A_2V_2 = \underline{0.0784 \text{ m}^3/\text{ s}}$.

c) $\frac{80\ 000}{9810}+4=\frac{V_2^2}{2g}+1.5\frac{V_2^2}{2g}.$ $\qquad \therefore V_2 = 9.77$ m / s. $\qquad \therefore Q=A_2V_2 = \underline{0.0767 \text{ m}^3/\text{ s}}$.

4.72 (**C**) Manometer: $g H+p_1=r g\frac{V_2^2}{2g}+\cancel{p_2}$ or $9810\times 0.02+p_1=r g\frac{V_2^2}{2g}.$

Energy: $K\frac{7.96^2}{2\times 9.81}=\frac{100000}{9810}.$ $\qquad \therefore K = 3.15.$

Combine the equations: $9810 \times 0.02 = 1.2 \times \frac{V_1^2}{2}$. $\quad \therefore V_1 = 18.1$ m/s.

4.73 Manometer: $gH + gz + p_1 = 13.6gH + gz + p_2$. $\quad \therefore \frac{p_1}{g} = 12.6H + \frac{p_2}{g}$.

Energy: $\frac{p_1}{g} + \frac{V_1^2}{2g} = \frac{p_2}{g} + \frac{V_2^2}{2g}$.

Combine energy and manometer: $12.6H = \frac{V_2^2 - V_1^2}{2g}$.

Continuity: $V_2 = \frac{d_1^2}{d_2^2} V_1$. $\quad \therefore V_1^2 = 12.6H \times 2g \Big/ \left(\frac{d_1^4}{d_2^4} - 1 \right)$.

$$\therefore Q = V_1 p \frac{d_1^2}{4} = \frac{p}{4} \left(\frac{12.6H \times 2g}{d_1^4 /\ d_2^4 - 1} \right)^{1/2} d_1^2 = \underline{12.35 d_1^2 d_2^2 \left(\frac{H}{d_1^4 - d_2^4} \right)^{1/2}}$$

4.74 Use the result of Problem 4.73:

a) $Q = 12.35 \times .16^2 \times .08^2 \left(\frac{.2}{.16^4 - .08^4} \right)^{1/2} = \underline{0.0365 \text{ m}^3/\text{s}}$.

b) $Q = 12.35 \times .24^2 \times .08^2 \left(\frac{.4}{.24^4 - .08^4} \right)^{1/2} = \underline{0.0503 \text{ m}^3/\text{s}}$.

c) Using English units with g = 32.2: $\quad Q = 22.37 d_1^2 d_2^2 \left(\frac{H}{d_1^4 - d_2^4} \right)^{1/2}$.

$$Q = 22.37 \times \left(\frac{1}{2} \right)^2 \left(\frac{1}{4} \right)^2 \left(\frac{10/\ 12}{.5^4 - .25^4} \right)^{1/2} = \underline{1.318 \text{ cfs}}.$$

d) $Q = 22.37 \times 1^2 \times \left(\frac{1}{3} \right)^2 \left(\frac{15/12}{1^4 - .3333^4} \right)^{1/2} = \underline{2.796 \text{ cfs}}$.

4.75 (**B**) $\quad h_L = K \frac{V^2}{2g} = \frac{\Delta p}{g}$. $\quad V = \frac{Q}{A} = \frac{0.040}{p \times 0.04^2} = 7.96$ m/s.

$$K \frac{7.96^2}{2 \times 9.81} = \frac{100000}{9810}. \quad \therefore K = 3.15.$$

4.76 a) Energy from surface to outlet: $\quad \frac{V_2^2}{2g} = H$. $\quad \therefore V_2^2 = 2gH$.

Energy from constriction to outlet: $\quad \frac{p_1}{g} + \frac{V_1^2}{2g} = \frac{p_2}{g} + \frac{V_2^2}{2g}$.

Continuity: $V_1 = 4V_2$. With $p_1 = p_v$ = 2450 Pa and p_2 = 100 000 Pa,

$$\frac{2450}{9810} + \frac{16}{2\times 9.81}\times 2gH = \frac{100\ 000}{9810} + \frac{1}{2\times 9.81}\times 2gH. \qquad \therefore H = \underline{0.663\text{ m}}.$$

b) With p_1 = 0.34 psia, p_2 = 14.7 psia,

$$\frac{.34\times 144}{62.4} + \frac{16}{2g}2gH = \frac{14.7\times 144}{62.4} + \frac{1}{2g}2gH. \qquad \therefore H = \underline{2.21\text{ ft}}.$$

4.77 Continuity: $V_1 = 4V_2$. Energy — surface to exit: $\frac{V_2^2}{2g} = H$.

Energy — constriction to exit: $\frac{p_v}{g} + \frac{V_1^2}{2g} = \frac{p_2}{g} + \frac{V_2^2}{2g}$.

$\therefore p_v = p_2 + \frac{V_2^2 - 16V_2^2}{2g}g = p_2 - 15Hg = 100\ 000 - 15\times .65\times 9810$ = 4350 Pa.

From Table B.1, T = $\underline{30°\text{C}}$.

4.78 Energy — surface to surface: $z_0 = z_2 + h_L$. $\therefore 30 = 20 + 2\frac{V_2^2}{2g}$.

Continuity: $V_1 = 4V_2$. $\therefore V_1^2 = 160\,g$. $\therefore V_2^2 = 10\,g$.

Energy — surface to constriction: $30 = \frac{160g}{2g} + \frac{(-94\ 000)}{9810} + z_1$

$\therefore z_1 = -40.4$ m. $\therefore H = 40.4 + 20 = \underline{60.4\text{ m}}$.

4.79 Continuity: $V_2 = \frac{10^2}{6^2}V_1 = 2.778\ V_1$.

Energy: $\frac{V_1^2}{2g} + \frac{p_1}{g} = \frac{V_2^2}{2g} + \frac{p_2}{g}$. $\frac{V_1^2}{2g} + \frac{200\ 000}{9810} = \frac{2.778^2 V_1^2}{2g} + \frac{2450}{9810}$.

$\therefore V_1 = 7.67$ m/s. $\therefore Q = p \times .05^2 \times 7.67 = \underline{0.0602\text{ m}^3/\text{s}}$.

4.80 Velocity at exit = V_e. Velocity in constriction = V_1. Velocity in pipe = V_2.

Energy — surface to exit: $\frac{V_e^2}{2g} = H$. $\therefore V_e^2 = 2gH$.

Continuity across nozzle: $V_2 = \frac{D^2}{d^2}V_e$. Also, $V_1 = 4V_2$.

Energy — surface to constriction: $H = \frac{V_1^2}{2g} + \frac{p_v}{g}$.

a) $5=\frac{1}{2g}\left(16\times\frac{D^4}{.2^4}\times 2g\times 5\right)+\frac{-97\ 550}{9810}.$ $\therefore D=\underline{0.131\text{ m}}$

b) $15=\frac{1}{2g}\left(16\frac{D^4}{(8/\ 12)^4}\times 2g\times 15\right)+\frac{(.34-14.7)144}{62.4}.$ $\therefore D=0.446'\text{ or }\underline{5.35''}$

4.81 Energy — surface to exit: $3=\frac{V_2^2}{2g}+4\frac{V_2^2}{2g}.$ $\therefore V_2^2=11.77.$

Energy — surface to "A": $3=\frac{11.77}{2\times 9.81}+\frac{1176-100\ 000}{9810}+(H+3)+1.5\frac{11.77^2}{2\times 9.81}.$

$\therefore H=\underline{8.57\text{ m}}.$

4.82 $\dot{m}=rAV=1.94\times p\times\left(\frac{1}{12}\right)^2\times 120=5.079\text{ slug/ sec.}$

$\dot{W}_P=5.079\times 32.2\left[\frac{30^2-120^2}{2\times 32.2}+\frac{120\times 144}{62.4}\right]/\,0.85=12{,}950\frac{\text{ft - lb}}{\text{sec}}$ or $\underline{23.5\text{ Hp}}.$

4.83 $\dot{m}=rAV=1000\times p\times .02^2\times 40=50.27\text{ kg / s.}$

$20\ 000=50.27\times 9.81\left[\frac{10^2-40^2}{2\times 9.81}+\frac{\Delta p}{9810}\right]/\ 0.82.$ $\therefore \Delta p=\underline{1.088\times 10^6\text{ Pa}}.$

4.84 (**C**) $\frac{\dot{W}_P}{gQ}=\cancel{\frac{V_2^2-V_1^2}{2g}}+\frac{\Delta p}{g}.$

$\dot{W}_P=Q\Delta p=0.040\times 400=16\text{ kW.}$ $\frac{\dot{W}_P}{h}=\frac{16}{0.89}=18.0\text{ kW.}$

4.85 $-\dot{W}_T=2\times 1000\times 9.81\left[\frac{0-10.2^2}{2\times 9.81}+\frac{-600\ 000}{9810}\right]\times 0.87.$ $\therefore \dot{W}_T=\underline{1.304\times 10^6\text{ W}}.$

We used $V_2=Q/\ A_2=\frac{2}{p\times .25^2}=10.2\text{ m / s.}$

4.86 $V_1=\frac{450}{p\times 3^2}=15.9\text{ fps.}$ $V_2=\frac{450}{p\times 3.75^2}=10.19\text{ fps.}$

$-10{,}000\left(\frac{1}{.746}\right)\times 550=450\times 1.94\times 32.2\left[\frac{10.19^2-15.9^2}{2\times 32.2}+\frac{(18-140)144}{62.4}\right]h_T.$

$\therefore h_T=\underline{0.924}$

4.87 a) $\dot{Q} - \dot{W}_S = \dot{m}g\left[\frac{V_2^2 - V_1^2}{2g} + \frac{p_2}{g_2} - \frac{p_1}{g_1} + z_2 - z_1 + \frac{c_v}{g}(T_2 - T_1)\right].$

The above is Eq. 4.5.17 with Eq. 4.5.18 and Eq. 1.7.13.

$$g_1 = \frac{p_1 g}{RT_1} = \frac{85 \times 9.81}{.287 \times 293} = 9.92\ \text{N / m}^3. \quad g_2 = \frac{600 \times 9.81}{.287\ T_2} = \frac{20\ 500}{T_2}.$$

$$\therefore -(-1\ 500\ 000) = 5 \times 9.81\left[\frac{200^2}{2 \times 9.81} + \frac{600\ 000 T_2}{20\ 500} - \frac{85\ 000}{9.92} + \frac{716.5}{9.81}(T_2 - 293)\right].$$

$\therefore T_2 = 572\ \text{K}$ or $\underline{299^\circ\text{C}}$.

Be careful of units! $p_2 = 600\ 000\ \text{Pa},\ c_v = 716.5\frac{\text{J}}{\text{K} \cdot \text{kg}}$

b) $-60\ 000 + 1\ 500\ 000$ = same as above. $\therefore T_2 = 560\ \text{K}$ or $\underline{287^\circ\text{C}}$.

4.88 $g_1 = \frac{p_1 g}{RT_1} = \frac{14.7 \times 144 \times 32.2}{1716 \times 520} = 0.0764\frac{\text{lb}}{\text{ft}^3}. \quad g_2 = \frac{60 \times 144 \times 32.2}{1716 \times 760} = 0.213\frac{\text{lb}}{\text{ft}^3}.$

$$c_v = 4296\frac{\text{ft - lb}}{\text{slug - }^\circ\text{R}}. \qquad \dot{m}g = rAVg = gAV = .213 \times p \times \left(\frac{1}{24}\right)^2 \times 600 = .697\ \text{lb / sec.}$$

Use Eq. 4.5.17 with Eqs. 4.5.18 and 1.7.13:

$$\dot{Q} + \dot{W}_c = \dot{m}g\left[\frac{V_2^2 - V_1^2}{2g} + \frac{p_2}{g_2} - \frac{p_1}{g_1} + \frac{c_v}{g}(T_2 - T_1) + z_2 - z_1\right].$$

$$-10 \times 778 \times .697 + \dot{W}_c = .697\left[\frac{600^2}{2 \times 32.2} + \frac{60 \times 144}{.213} - \frac{14.7 \times 144}{.0764} + \frac{4296}{32.2}(300 - 60)\right]$$

$\therefore \dot{W}_c = 40\ 600\frac{\text{ft - lb}}{\text{sec}}$ or $\underline{73.8\ \text{Hp}}$.

4.89 Energy — surface to exit: $-\dot{W}_T h_T = \dot{m}g\left[\frac{V_2^2}{2g} - 20 + 4.5\frac{V_2^2}{2g}\right].$

$$V_2 = \frac{15}{p \times .6^2} = 13.26\ \text{m / s.} \qquad \dot{m}g = Qg = 15 \times 9810 = 147\ 150\ \text{N / s.}$$

$$-\dot{W}_T \times 0.8 = 147\ 150\left[\frac{13.26^2}{2 \times 9.81} - 20 + 4.5\frac{13.26^2}{2 \times 9.81}\right]. \qquad \therefore \dot{W}_T = \underline{5390\ \text{kW}}.$$

4.90 (**D**) $36.0 + 15 = \frac{4.58^2}{2 \times 9.81} + \frac{p_B}{9810} + 3.2\frac{7.16^2}{2 \times 9.81}. \quad \therefore p_B = \underline{416\ 000\ \text{Pa}}$

In the above energy equation we used

$$h_L = K\frac{V^2}{2g} \quad \text{with} \quad V = \frac{Q}{A} = \frac{0.2}{p \times 0.2^2} = 4.42\ \text{m/s.}$$

4.91 Energy — surface to “C”:

$$\dot{W}_P \times .8 + \dot{m}g \times 10 = \left[\frac{10^2}{2\times 9.81} + \frac{200\ 000}{9810} + 7.7\frac{10^2}{2\times 9.81}\right]770.5.$$

$(\dot{m}g = \rho AVg = 1000\times \pi \times .05^2 \times 10\times 9.81 = 770.5 \text{ N/ s.})$ $\quad \therefore \dot{W}_P = \underline{52\ 700 \text{ W}}.$

Energy — surface to “A”: $30 = \dfrac{10^2}{2\times 9.81} + \dfrac{p_A}{9810} + 1.5\dfrac{10^2}{2\times 9.81}. \quad \therefore p_A = \underline{169\ 300 \text{ Pa}}.$

Energy — surface to “B”: $\dot{W}_P \eta_P = \dot{m}g\left[\dfrac{V_B^2 - V_O^2}{2g} + \dfrac{p_B - p_O}{\gamma} + z_B - z_O + K\dfrac{V_B^2}{2g}\right]$

$$52\ 700\times .8 = 770.5\left[\frac{10^2}{2\times 9.81} + \frac{p_B}{9810} - 30 + 1.5\frac{10^2}{2\times 9.81}\right]. \quad \therefore p_B = \underline{706\ 100 \text{ Pa}}.$$

4.92 Manometer: $\gamma \times \dfrac{20}{12} + \gamma z_1 + p_1 = 13.6\gamma \times \dfrac{20}{12} + \gamma z_2 + p_2 + \dfrac{V_2^2}{2}\rho.$

$$\therefore \frac{20}{12} + z_1 + \frac{p_1}{\gamma} = 13.6\times \frac{20}{12} + z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g}.$$

Energy: $\dfrac{V_1^2}{2g} + z_1 + \dfrac{p_1}{\gamma} = H_T + z_2 + \dfrac{p_2}{\gamma} + \dfrac{V_2^2}{2g}.$

$$\therefore \frac{20}{12} = 13.6\times \frac{20}{12} + \frac{V_1^2}{2g} - H_T. \quad V_1 = \frac{18}{\pi \times \left(\frac{1}{3}\right)^2} = 51.6 \text{ fps}.$$

$$\therefore H_T = 12.6\times \frac{20}{12} + \frac{51.6^2}{2\times 32.2} = 62.3'. \quad \dot{W}_T = \gamma Q \eta_T H_T = 62.4\times 18\times .9\times 62.3$$

$$= 62{,}980 \ \frac{\text{ft - lb}}{\text{sec}} \quad \text{or} \quad \underline{115 \text{ Hp}}.$$

4.93 Energy—across the nozzle: $\dfrac{p_1}{\gamma} + \dfrac{V_1^2}{2g} = \dfrac{p_2}{\gamma} + \dfrac{V_2^2}{2g}. \quad V_2 = \dfrac{5^2}{2^2}V_1 = 6.25V_1.$

$$\therefore \frac{400\ 000}{9810} + \frac{V_1^2}{2\times 9.81} = \frac{6.25^2 V_1^2}{2\times 9.81}. \quad \therefore V_1 = 4.58 \text{ m/s},\ V_A = 7.16 \text{ m/s},\ V_2 = 28.6 \text{ m/s}.$$

Energy—surface to exit:

$$H_P + 15 = \frac{28.6^2}{2\times 9.81} + 1.5\frac{4.58^2}{2\times 9.81} + 3.2\frac{7.16^2}{2\times 9.81}. \quad \therefore H_P = 36.8 \text{ m}.$$

$$\therefore \dot{W}_P = \gamma Q H_P / \eta_P = 9810\times (\pi \times .01^2)\times 28.6\times 36.8/.85 = \underline{3820 \text{ W}}.$$

Energy —surface to "A":

$$15 = \frac{7.16^2}{2\times 9.81} + \frac{p_A}{9810} + 3.2\frac{7.16^2}{2\times 9.81}. \qquad \therefore\ p_A = \underline{39\ 400\ \text{Pa}}$$

Energy —surface to "B":

$$36.0 + 15 = \frac{4.58^2}{2\times 9.81} + \frac{p_B}{9810} + 3.2\frac{7.16^2}{2\times 9.81}. \qquad \therefore\ p_B = \underline{416\ 000\ \text{Pa}}$$

4.94 (**A**) $V = \frac{Q}{A} = \frac{0.1}{p\times .04^2} = 19.89\ \text{m / s}.$

Energy —surface to entrance: $H_P = \frac{V_2^2}{2g} + \frac{p_2}{g} + z_2 + K\frac{V_2^2}{2g}.$

$$\therefore H_P = \frac{19.89^2}{2\times 9.81} + \frac{180\ 000}{9810} + 50 + 5.6\frac{19.89^2}{2\times 9.81} = 201.4\ \text{m}.$$

$$\therefore \dot{W}_P = gQH_P / h_P = 9810\times 0.1\times 201.4 / 0.75 = \underline{263\ 000\ \text{W}}.$$

4.95 Energy —surface to exit: $10 = \frac{V_2^2}{2g} + \frac{p_2}{g} + z_2 + 2.2\frac{V_2^2}{2g}.$

$\therefore V_2 = 7.83\ \text{m / s}.$ $Q = 0.02 = 7.83\times p d_2^2 / 4.$ $\therefore d_2 = \underline{0.0570\ \text{m}}.$

4.96 Depth on raised section = y_2. Continuity: $3\times 3 = V_2 y_2$.

Energy (see Eq. 4.5.21): $\frac{3^2}{2g} + 3 = \frac{V_2^2}{2g} + (0.4 + y_2).$

$\therefore 3.059 = \frac{9^2}{2g\,y_2^2} + y_2,$ or $y_2^3 - 3.059\,y_2^2 + 4.128 = 0$

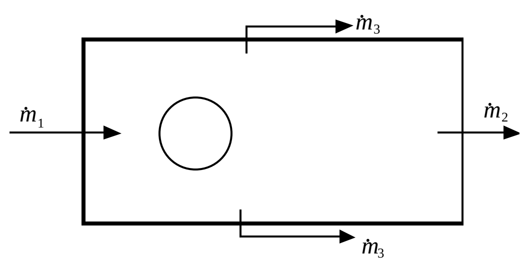
Trial-and-error:

$$\left.\begin{array}{ll} y_2 = 2.0: & -.11 \overset{?}{=} 0. \\ y_2 = 1.8: & +.05 \overset{?}{=} 0. \end{array}\right\} \quad \therefore y_2 = \underline{1.85\ \text{m}}.$$

$$\left.\begin{array}{ll} y_2 = 2.1: & -.1 \overset{?}{=} 0. \\ y_2 = 2.3: & +.1 \overset{?}{=} 0. \end{array}\right\} \quad \therefore y_2 = \underline{2.22\ \text{m}}.$$

The depth that actually occurs depends on the downstream conditions. We cannot select a "correct" answer between the two.

4.97 Mass flux occurs as shown. The velocity of all fluid elements leaving the top and bottom is approximately 32 m/s. The distance where $u = 32$ m / s is $y = \pm 2$ m.



To find $\dot{m}_3$ use continuity:

$$\dot{m}_1 = \dot{m}_2 + 2\dot{m}_3. \qquad r4\times 10\times 32 = r2\int_0^2 (28+y^2)10dy + 2\dot{m}_3.$$

$$\therefore \dot{m}_3 = 640r - 10r\left[28\times 2+\frac{8}{3}\right] = 53.3r.$$

Rate of K.E. loss = $\dot{m}_1 \frac{V_1^2}{2} - 2\dot{m}_3 \frac{V_1^2}{2} - r2\int_0^2 \frac{u^3}{2}10dy$

$$= 1280r\frac{32^2}{2} - 53.3r32^2 - 10r\int_0^2 (28+y^2)^3 dy$$

$$= r[655360 - 54579 - 507320] = \underline{115000\ W}.$$

4.98 The average velocity at section 2 is also 8 m / s. The kinetic-energy-correction factor for a parabola is 2 (see Example 4.9). The energy equation is:

$$\frac{V_1^2}{2g} + \frac{p_1}{g} = a_2\frac{\overline{V}_2^2}{2g} + \frac{p_2}{g} + h_L.$$

$$\frac{8^2}{2\times 9.81} + \frac{150\ 000}{9810} = 2\frac{8^2}{2\times 9.81} + \frac{110\ 000}{9810} + h_L.$$

$$\therefore h_L = \underline{0.815\text{ m}}.$$

4.99 $\overline{V} = \frac{1}{A}\int VdA = \frac{1}{2}\int_0^2 (28+y^2)dy = \frac{1}{2}\left[28\times 2+\frac{2^3}{3}\right] = 29.33\text{ m/ s}$

$$a = \frac{1}{A\overline{V}^3}\int V^3 dA = \frac{1}{2\times 29.33^3}\int_0^2 (28+y^2)^3 dy$$

$$= \frac{1}{2\times 29.33^3}\left[28^3\times 2 + 3\times 28^2\times 2^3/\ 3 + 3\times 28\times 2^5/\ 5 + 2^7/\ 7\right] = \underline{1.005}$$

4.100 a) $\overline{V} = \frac{1}{A}\int VdA = \frac{1}{p\times 0.01^2}\int_0^{0.01} 10\left(1-\frac{r^2}{0.01^2}\right)2p\,rdr = \frac{20}{0.01^2}\left[\frac{0.01^2}{2} - \frac{0.01^4}{4\times 0.01^2}\right] = 5\text{ m/s}$

$$a = \frac{1}{A\overline{V}^3}\int V^3 dA = \frac{1}{p\times 0.01^2\times 5^3}\int_0^{0.01} 10^3\left(1-\frac{r^2}{0.01^2}\right)^3 2prdr$$

$$= \frac{2000}{0.01^2\times 5^3}\left(\frac{0.01^2}{2} - \frac{3\times 0.01^4}{4\times 0.01^2} + \frac{3\times 0.01^6}{6\times 0.01^4} - \frac{0.01^8}{8\times 0.01^6}\right) = \underline{2.00}$$

b) $\overline{V} = \frac{1}{A}\int VdA = \frac{1}{0.02w}\int_0^{0.02} 10\left(1 - \frac{y^2}{0.02^2}\right)wdy = \frac{10}{0.02}\left(0.02 - \frac{0.02^3}{3\times 0.02^2}\right) = 6.67 \text{ m/s}$

$$a = \frac{1}{A\overline{V}^3}\int V^3 dA = \frac{1}{0.02\times w\times 6.67^3}\int_0^{0.02} 10^3\left(1 - \frac{y^2}{0.02^2}\right)^3 wdy$$

$$= \frac{1000}{0.02\times 6.67^3}\left(0.02 - \frac{3\times 0.02^3}{3\times 0.02^2} + \frac{3\times 0.02^5}{5\times 0.02^4} - \frac{0.02^7}{7\times 0.02^6}\right) = \underline{1.541}$$

4.101 $\overline{V} = \frac{1}{A}\int VdA = \frac{1}{pR^2}\int_0^R u_{\max}\left(1 - \frac{r}{R}\right)^{1/n} 2prdr = -2u_{\max}\left(\frac{n}{2n+1} - \frac{n}{n+1}\right)$

$$K.E. = r\int V\left(\frac{V^2}{2}\right)dA = \frac{r}{2}\int_0^R u_{\max}^3\left(1 - \frac{r}{R}\right)^{3/n} 2prdr = rpu_{\max}^3\left[-R^2\left(\frac{n}{3+2n} - \frac{n}{3+n}\right)\right]$$

a) $\overline{V} = -2u_{\max}\left(\frac{5}{11} - \frac{5}{6}\right) = 0.758\ u_{\max}$

$$K.E. = rpR^2u_{\max}^3\left(\frac{5}{8} - \frac{5}{13}\right) = \underline{0.24\ rpR^2u_{\max}^3}$$

$$a = \frac{K.E.}{\frac{1}{2}rA\overline{V}^3} = \frac{0.24\,rpR^2u_{\max}^3}{\frac{1}{2}rpR^2\times 0.758^3u_{\max}^3} = \underline{1.102}$$

b) $\overline{V} = -2u_{\max}\left(\frac{7}{15} - \frac{7}{8}\right) = 0.817\ u_{\max}$

$$K.E. = rpu_{\max}^3R^2\left(\frac{7}{10} - \frac{7}{17}\right) = \underline{0.288\ rpR^2u_{\max}^3}$$

$$a = \frac{K.E.}{rA\overline{V}\left(\frac{\overline{V}^2}{2}\right)} = \frac{0.288\,rpR^2u_{\max}^3}{rpR^2\times 0.817u_{\max}\left(\frac{0.817^2u_{\max}^2}{2}\right)} = \underline{1.056}$$

c) $\overline{V} = -2u_{\max}\left(\frac{9}{19} - \frac{9}{10}\right) = 0.853\ u_{\max}$

$$K.E. = rpR^2u_{\max}^3\left(\frac{9}{12} - \frac{9}{21}\right) = \underline{0.321\ rpR^2u_{\max}^3}$$

$$a = \frac{K.E.}{\frac{1}{2}rA\overline{V}^3} = \frac{0.321rpR^2u_{\max}^3}{\frac{1}{2}rpR^2\times 0.853^3u_{\max}^3} = \underline{1.034}$$

4.102 Engine power = $F_D \times V_\infty + \dot{m}\left(\frac{V_2^2 - V_1^2}{2} + \tilde{u}_2 - \tilde{u}_1\right)$

$$\dot{m}_f g_f = F_D V_\infty + \dot{m}\left[\frac{V_2^2 - V_1^2}{2} + c_v(T_2 - T_1)\right]$$

4.103

$$\dot{W}h = F_D \times V$$

$$\frac{10^{-3}}{5}\left(\frac{\text{m}^3}{\text{km}}\right)\times 930\left(\frac{\text{kg}}{\text{m}^3}\right)\times q_f\left(\frac{\text{kJ}}{\text{kg}}\right)\times\frac{100}{3600}\left(\frac{\text{km}}{\text{s}}\right)\times 0.15 = \frac{1340}{1000}\times\frac{100\ 000}{3600}$$

$$\therefore q_f = \underline{48\ 030\ \text{kJ/kg}}$$

4.104 $0 = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{g} + z_2 - \frac{V_1^2}{2g} - \frac{p_1}{g} - z_1 + 32\frac{nLV}{gD^2}$

$$0 = 2\frac{V^2}{2\times 9.81} - 0.35 + 32\times\frac{10^{-6}\times 180V}{9.81\times 0.02^2}.$$

$V^2 + 14.4V - 3.434 = 0.$ $\therefore V = 0.235$ m/s and $Q = \underline{7.37\times 10^{-5}\ \text{m}^3/\text{s}}$

4.105 Energy from surface to surface: $H_P = \frac{V_2^2}{2g} + \frac{p_2}{g} + z_2 - \frac{V_1^2}{2g} - \frac{p_1}{g} - z_1 + K\frac{V^2}{2g}.$

a) $H_P = 40 + 5\frac{Q^2}{p\times 0.04^2\times 2\times 9.81} = 40 + 50.7\ Q^2$

Try $Q = 0.25$: $H_P = 43.2$ (energy). $H_P = 58$ (curve)

Try $Q = 0.30$: $H_P = 44.6$ (energy). $H_P = 48$ (curve)

Solution: $Q = \underline{0.32\ \text{m}^3/\text{s}}$.

b) $H_P = 40 + \frac{20\ Q^2}{p\times 0.04^2\times 2\times 9.81} = 40 + 203\ Q^2$

Try $Q = 0.25$: $H_P = 52.7$ (energy). $H_P = 58$ (curve)

Solution: $Q = \underline{0.27\ \text{m}^3/\text{s}}$

Note: The curve does not allow for significant accuracy.

4.106 Continuity: $A_1 V_1 = A_2 V_2 + A_3 V_3$

$$p\times 0.06^2\times 5 = p\times 0.02^2\times 20 + p\times 0.03^2 V_3. \qquad \therefore V_3 = 11.11\ \text{m/s}$$

Energy: energy in + pump energy = energy out

$$\dot{m}_1\left(\frac{V_1^2}{2} + \frac{p_1}{r}\right) + \dot{W}_P\times h_P = \dot{m}_2\left(\frac{V_2^2}{2} + \frac{p_2}{r}\right) + \dot{m}_3\left(\frac{V_3^2}{2} + \frac{p_3}{r}\right)$$

$$1000\pi \times 0.06^2 \times 5\left(\frac{5^2}{2}+\frac{120\ 000}{1000}\right)+0.85\dot{W}_P = 1000\pi \times 0.02^2 \times 20\left(\frac{20^2}{2}+\frac{300\ 000}{1000}\right)$$

$$+1000\pi \times 0.03^2 \times 11.11\left(\frac{11.11^2}{2}+\frac{500\ 000}{1000}\right)$$

$$\therefore \dot{W}_P = \underline{26\ 700\ \text{W}}$$

4.107 (**A**) After the pressure is found, that pressure is multiplied by the area of the window. The pressure is relatively constant over the area.

4.108 $\frac{V_1^2}{2g}+\frac{p_1}{g}=\frac{V_2^2}{2g}+\frac{p_2}{g}.$ $\qquad V_2=\frac{d^2}{(d/2)^2}V_1=4\ V_1.$

a) $\frac{V_1^2}{2\times 9.81}+\frac{200\ 000}{9810}=\frac{16\ V_1^2}{2\times 9.81}.$ $\quad \therefore V_1 = 5.164\ \text{m/s}.$

$p_1A_1 - F = \dot{m}(V_2 - V_1).$

$200\ 000\pi\times.03^2 - F = 1000\pi\times.03^2\times 5.164(4\times 5.164-5.164).$ $\quad \therefore F = \underline{339\ \text{N}}.$

b) $\frac{V_1^2}{2\times 9.81}+\frac{400\ 000}{9810}=\frac{16\ V_1^2}{2\times 9.81}.$ $\quad \therefore V_1 = 7.303\ \text{m/s}.$

$400\ 000\pi\times.03^2 - F = 1000\pi\times.03^2\times 7.303(4\times 7.303-7.303).$ $\quad \therefore F = \underline{679\ \text{N}}.$

c) $\frac{V_1^2}{2\times 9.81}+\frac{200\ 000}{9810}=\frac{16\ V_1^2}{2\times 9.81}.$ $\quad \therefore V_1 = 5.164\ \text{m/s}.$

$200\ 000\pi\times.06^2 - F = 1000\pi\times.06^2\times 5.164(4\times 5.164-5.164).$ $\quad \therefore F = \underline{1356\ \text{N}}.$

d) $\frac{V_1^2}{2\times 32.2}+\frac{30\times 144}{62.4}=\frac{16\ V_1^2}{2\times 32.2}.$ $\quad \therefore V_1 = 17.24\ \text{fps}.$

$30\times\pi\times 1.5^2 - F = 1.94\times\pi\times(1.5/12)^2\times 17.24^2(4-1).$ $\quad \therefore F = \underline{127\ \text{lb}}.$

e) $\frac{V_1^2}{2\times 32.2}+\frac{60\times 144}{62.4}=\frac{16\ V_1^2}{2\times 32.2}.$ $\quad \therefore V_1 = 24.38\ \text{fps}.$

$60\times\pi\times 1.5^2 - F = 1.94\times\pi\times(1.5/12)^2\times 24.38^2(4-1).$ $\quad \therefore F = \underline{254\ \text{lb}}.$

f) $\frac{V_1^2}{2\times 32.2}+\frac{30\times 144}{62.4}=\frac{16\ V_1^2}{2\times 32.2}.$ $\quad \therefore V_1 = 17.24\ \text{fps}.$

$30\times\pi\times 3^2 - F = 1.94\times\pi\times(3/12)^2\times 17.24^2(4-1).$ $\quad \therefore F = \underline{509\ \text{lb}}.$

4.109 $\frac{V_1^2}{2g}+\frac{p_1}{g}=\frac{V_2^2}{2g}+\frac{p_2}{g}.$ $\qquad V_2=\frac{9^2}{3^2}V_1=9V_1.$

$\frac{V_1^2}{2\times 9.81}+\frac{2\ 000\ 000}{9810}=\frac{81\ V_1^2}{2\times 9.81}.$ $\qquad \therefore V_1^2 = 50.$

$$p_1A_1 - F = \dot{m}(V_2 - V_1) = \dot{m}8V_1$$
$$2\ 000\ 000p\times.045^2 - F = 1000p\times.045^2\times 8\times 50$$
$$\therefore F = \underline{10\ 180\ \text{N}}.$$

4.110 $\frac{V_1^2}{2g}+\frac{p_1}{g}=\frac{V_2^2}{2g}+\frac{p_2}{g}$. $V_0p\times.01^2 = V_e\times.006\times.15$. $\therefore V_e = 11.1$ m/s.

$\Sigma F_x = \dot{m}(V_{2x} - V_{1x})$.

a) $V_2 = \frac{10^2}{8^2}V_1 = 1.562\ V_1$. $\frac{V_1^2}{2\times 9.81}+\frac{400\ 000}{9810}=\frac{2.441\ V_1^2}{2\times 9.81}$. $\therefore V_1 = 23.56$ m / s.

$\therefore p_1A_1 - F = \dot{m}(V_2 - V_1)$.

$400\ 000p\times.05^2 - F = 1000p\times.05^2\times 23.56(.562\times 23.56)$. $\therefore F = \underline{692\ \text{N}}$.

b) $V_2 = \frac{10^2}{6^2}V_1 = 2.778\ V_1$. $\frac{V_1^2}{2g}+\frac{400\ 000}{9810}=\frac{7.716\ V_1^2}{2g}$. $\therefore V_1 = 10.91$ m / s.

$400\ 000p\times.05^2 - F = 1000p\times.05^2\times 10.91(1.778\times 10.91)$. $\therefore F = \underline{1479\ \text{N}}$.

c) $V_2 = \frac{10^2}{4^2}V_1 = 6.25\ V_1$. $\frac{V_1^2}{2g}+\frac{400\ 000}{9810}=\frac{39.06\ V_1^2}{2g}$. $\therefore V_1 = 4.585$ m / s.

$400\ 000p\times.05^2 - F = 1000p\times.05^2\times 4.585(5.25\times 4.585)$. $\therefore F = \underline{2275\ \text{N}}$.

d) $V_2 = \frac{10^2}{2^2}V_1 = 25\ V_1$. $\frac{V_1^2}{2g}+\frac{400\ 000}{9810}=\frac{625\ V_1^2}{2g}$. $\therefore V_1 = 1.132$ m / s.

$400\ 000p\times.05^2 - F = 1000p\times.05^2\times 1.132(24\times 1.132)$. $\therefore F = \underline{2900\ \text{N}}$.

4.111 (**C**) $\frac{V_1^2}{2g}+\frac{p_1}{g}=\frac{V_2^2}{2g}+\cancel{\frac{p_2}{g}}$. $p_1 = 9810\times\frac{(6.25^2-1)\times 12.73^2}{2\times 9.81} = 3085000$ Pa.

$p_1A_1 - F = rQ(V_2 - V_1)$. $3085000\times p\times 0.05^2 - F = 1000\times 0.1\times 12.73(6.25-1)$

$$\therefore F = \underline{17500\ \text{N}}.$$

4.112 $V_2 = 4V_1 = 120$ fps. $p_1 = g\left[\frac{V_2^2 - V_1^2}{2g}\right] = 62.4\left[\frac{120^2 - 30^2}{2\times 32.2}\right] = 13{,}080$ psf.

$$F = p_1A_1 - \dot{m}(V_{2x} - V_{1x}) = 13{,}080\ p\left(\frac{1.5}{12}\right)^2 - 1.94\times p\left(\frac{1.5}{12}\right)^2\times 30(-120-30) = \underline{1072\ \text{lb}}.$$

4.113 $V_2 = 4\ V_1$. $\frac{V_1^2}{2g}+\frac{p_1}{g}=\frac{V_2^2}{2g}+\frac{p_2}{g}$. $\therefore \frac{15\ V_1^2}{2g}=\frac{p_1}{g}$.

a) $V_1^2 = \frac{2\times 9.81}{15\times 9810}\times 200\ 000 = 26.67$. $\therefore V_1 = 5.16$ m / s, $V_2 = 20.7$ m / s.

$p_1A_1 - F_x = \dot{m}(V_{2x} - V_{1x})$. $\therefore F_x = 200\ 000p\times.04^2 + 1000p\times.04^2\times 5.16^2 = \underline{1139\ \text{N}}$.

$$F_y = \dot{m}(V_{2y} - V_{1y}). \qquad \therefore F_y = 1000p \times .04^2 \times 5.16(20.7) = \underline{537\text{ N}}.$$

b) $V_1^2 = \dfrac{2 \times 9.81}{15 \times 9810} \times 400\ 000 = 53.33. \qquad \therefore V_1 = 7.30\text{ m/ s}, \ \ V_2 = 29.2\text{ m/ s}.$

$$p_1 A_1 - F_x = \dot{m}(V_{2x} - V_{1x}). \qquad \therefore F_x = 400\ 000p \times .04^2 + 1000p \times .04^2 \times 7.3^2 = \underline{2280\text{ N}}.$$

$$F_y = \dot{m}(V_{2y} - V_{1y}) = 1000p \times .04^2 \times 7.3 \times (29.2) = \underline{1071\text{ N}}.$$

c) $V_1^2 = \dfrac{2 \times 9.81}{15 \times 9810} \times 800\ 000 = 106.7. \qquad \therefore V_1 = 10.33\text{ m/ s}, \ \ V_2 = 41.3\text{ m/ s}.$

$$F_x = p_1 A_1 + r A_1 V_1^2 = 800\ 000p \times .04^2 + 1000p \times .04^2 \times 10.33^2 = \underline{4560\text{ N}}.$$

$$F_y = \dot{m}(V_{2y}) = 1000p \times .04^2 \times 10.33(41.3) = \underline{2140\text{ N}}.$$

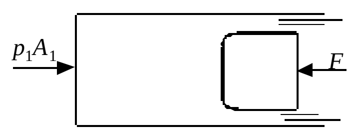
4.114 $V_2 = \dfrac{40^2}{10^2} V_1 = 80\text{ m/ s}. \qquad \dfrac{V_1^2}{2g} + \dfrac{p_1}{g} = \dfrac{V_2^2}{2g} + \dfrac{p_2}{g}$

$$\therefore p_1 = 9810\left[\frac{80^2}{2 \times 9.81} - \frac{5^2}{2 \times 9.81}\right] = 3.19 \times 10^6\text{ Pa}.$$

$$p_1 A_1 - F = \dot{m}(V_{2x} - V_{1x}). \ \therefore F = 3.19 \times 10^6 p \times .2^2 - 1000p \times .2^2 \times 5(80 - 5) = \underline{353\ 000\text{ N}}.$$

4.115 $A_1 V_1 = A_2 V_2. \qquad p \times .025^2 \times 4 = p(.025^2 - .02^2)V_2.$

$\therefore V_2 = 11.11\text{ m/ s}. \qquad \dfrac{p_1}{g} + \dfrac{V_1^2}{2g} = \dfrac{p_2}{g} + \dfrac{V_2^2}{2g}.$



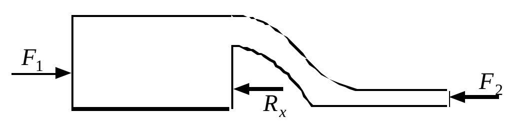
$$p_1 = 9810\left(\frac{11.11^2 - 4^2}{2 \times 9.81}\right) = 53700\text{ Pa}.$$

$$p_1 A_1 - F = \dot{m}(V_2 - V_1).$$

$$\therefore F = 53\ 700p \times .025^2 - 1000p \times .025^2 \times 4(11.11 - 4) = \underline{49.6\text{ N}}.$$

4.116 Continuity: $.7\ V_1 = .1\ V_2. \qquad \therefore V_2 = 7\ V_1.$

Energy: $\dfrac{V_1^2}{2g} + \dfrac{p_1}{g} + z_1 = \dfrac{V_2^2}{2g} + \dfrac{p_2}{g} + z_2$



$$\frac{V_1^2}{2 \times 9.81} + .7 = \frac{49V_1^2}{2 \times 9.81} + .1. \qquad \therefore V_1 = 0.495, \quad V_2 = 3.467\text{ m/ s}.$$

Momentum: $F_1 - F_2 - R_x = \dot{m}(V_2 - V_1)$

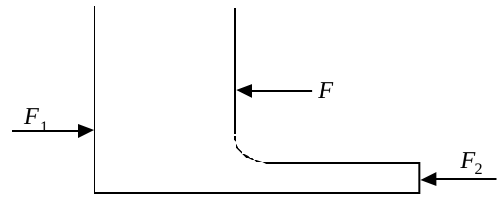
$$9810 \times .35(.7 \times 1.5) - 9810 \times .05(0.1 \times 1.5) - R_x = 1000 \times (.1 \times 1.5) \times 3.467(3.467 - .495)$$

$$\therefore R_x = \underline{1986\text{ N}}.$$

$\therefore R_x$ acts to the left on the water, and to the right on the obstruction.

4.117 Continuity: $6\ V_1 = .2\ V_2$. $\therefore V_2 = 30\ V_1$.

Energy (along bottom streamline):



$$\frac{V_1^2}{2g} + \frac{p_1}{g} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{g} + z_2$$

$$\frac{V_2^2/900}{2\times 9.81} + 6 = \frac{V_2^2}{2\times 9.81} + 0.2.$$

$\therefore V_2 = 10.67$, $V_1 = .36$ m/s.

Momentum: $F_1 - F_2 - F = \dot{m}(V_2 - V_1)$

$$9810\times 3(6\times 4) - 9810\times .1(.2\times 4) - F = 1000\times(.2\times 4)\times 10.67(10.67 - .36)$$

$\therefore F = \underline{618\ 000\ \text{N}}$. ($F$ acts to the right on the gate.)

4.118 a) $8\times .6 = V_2 y_2$. $F_1 - F_2 = \dot{m}(V_2 - V_1)$.

$$g\times .3\times .6w - g\frac{y_2}{2}y_2 w = r.6w\times 8\left(\frac{8\times .6}{y_2} - 8\right).$$

$$\frac{g}{2}(.36 - y_2^2) = 4.8r\times 8\frac{.6 - y_2}{y_2}. \qquad \therefore (.6 + y_2)y_2 = \frac{4.8\times 8\times 2}{9.81}.$$

$y_2^2 + .6y_2 - 7.829 = 0$. $\therefore y_2 = \underline{2.51\ \text{m}}$. (See Example 4.12.)

b) $y_2 = \frac{1}{2}\left[-y_1 + \sqrt{y_1^2 + \frac{8}{g}y_1 V_1^2}\right] = \frac{1}{2}\left[-.4 + \sqrt{.4^2 + \frac{8}{9.81}\times .4\times 12^2}\right] = \underline{3.23\ \text{m}}$.

c) $y_2 = \frac{1}{2}\left[-y_1 + \sqrt{y_1^2 + \frac{8}{g}y_1 V_1^2}\right] = \frac{1}{2}\left[-2 + \sqrt{2^2 + \frac{8}{32.2}\times 2\times 20^2}\right] = \underline{6.12\ \text{ft}}$.

d) $y_2 = \frac{1}{2}\left[-y_1 + \sqrt{y_1^2 + \frac{8}{g}y_1 V_1^2}\right] = \frac{1}{2}\left[-3 + \sqrt{3^2 + \frac{8}{32.2}\times 3\times 30^2}\right] = \underline{11.54\ \text{ft}}$.

4.119 Continuity: $V_2 y_2 = V_1 y_1 = 4V_2 y_1$. $\therefore y_2 = 4y_1$.

Use the result of Example 4.12: $y_2 = \frac{1}{2}\left[-y_1 + \left(y_1^2 + \frac{8}{g}y_1 V_1^2\right)^{1/2}\right]$

a) $y_2 = 4\times .8 = \underline{3.2\ \text{m}}$.

$$3.2 = \frac{1}{2}\left[-.8 + \left(.8^2 + \frac{8}{9.81}\times .8\times V_1^2\right)^{1/2}\right]. \qquad \therefore V_1 = \underline{8.86\ \text{m/ s}}.$$

b) $y_2 = 4\times 2 = \underline{8\ \text{ft}}$.

$$8 = \frac{1}{2}\left[-2 + \left(2^2 + \frac{8}{32.2}\times 2\times V_1^2\right)^{1/2}\right]. \qquad \therefore V_1 = \underline{25.4\ \text{fps}}.$$

4.120 $V = \frac{9}{3\times 3} = 1\text{ m / s}.$ $\frac{1^2}{2\times 9.81} + 3 = \frac{V_1^2}{2\times 9.81} + y_1.$ $V_1 y_1 = 1\times 3.$

$\therefore 3.05 = \frac{V_1^2}{19.62} + \frac{3}{V_1}.$ Trial-and-error: $V_1 = 7: \quad 3.05 \overset{?}{=} 2.93$; $V_1 = 7.2: \quad 3.05 \overset{?}{=} 3.06$ $\Big\}$ $V_1 = 7.19\text{ m / s}.$ $y_1 = .417\text{ m}.$

$$y_2 = \frac{1}{2}\left[-.417 + \left(.417^2 + \frac{8}{9.81}\times .417\times 7.19^2\right)^{1/2}\right] = \underline{1.90\text{ m}}.$$

$V_2 \times 1.9 = 7.19 \times .417.$ $V_2 = \underline{1.58\text{ m / s}}.$

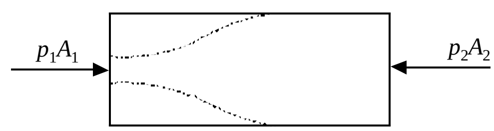
4.121 Refer to Example 4.12: $g\frac{y_1}{2}y_1 w - g \times 3\times 6w = r\times 6w\times 10\left(10 - \frac{60}{y_1}\right).$ $(V_1 y_1 = 6\cdot 10).$

$\therefore \frac{g}{2}(y_1^2 - 36) = 600r\left(\frac{y_1 - 6}{y_1}\right).$ $\therefore (y_1 + 6)y_1 = \frac{1200}{32.2} = 37.27.$ $\therefore y_1 = \underline{3.8\text{ ft}},$ $V_1 = \underline{15.8\text{ fps}}.$

4.122 Continuity: $20\times p\times .015^2 = V_2 p\times .03^2.$

$\therefore V_2 = 5\text{ m / s}.$

Momentum: $p_1 A_1 - p_2 A_2 = \dot{m}(V_2 - V_1).$



$$60\ 000p\times .03^2 - p_2 p\times .03^2 = 1000p\times .015^2\times 20(5-20). \quad \therefore p_2 = \underline{135\text{ kPa}}.$$

4.123 $V_1 A_1 = 2V_2 A_2.$ $V_2 = 15\frac{p\times .05^2}{2p\times .025^2} = 30\text{ m/s}.$

$\frac{p_1}{g} + \frac{V_1^2}{2g} = \frac{p_2}{g} + \frac{V_2^2}{2g}.$ $\therefore p_1 = 9810\frac{30^2 - 15^2}{2\times 9.81} = 337\ 500\text{ Pa}.$

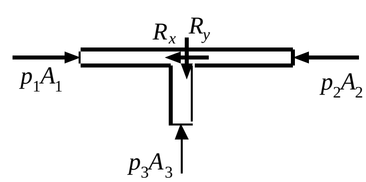
$\Sigma F_x = \dot{m}\left(V_{2x} - V_{1x}\right).$ $p_1 A_1 - F = \dot{m}(-V_1).$

$\therefore F = p_1 A_1 + \dot{m}V_1 = 337\ 500p\times .05^2 + 1000p\times .05^2\times 15^2 = \underline{4420\text{ N}}.$

4.124 $\dot{m}_1 = 1000p\times .03^2\times 12 = 33.93\text{ kg / s}.$

$\dot{m}_3 = 1000p\times .02^2\times 8 = 10.05\text{ kg / s}.$

$\therefore \dot{m}_2 = \dot{m}_1 - \dot{m}_3 = 23.88 = 1000p\times .03^2 V_2.$ $\therefore V_2 = 8.446\text{ m / s}.$



Energy from $1 \to 2$: $\frac{V_1^2}{2g} + \frac{p_1}{g} = \frac{V_2^2}{2g} + \frac{p_2}{g}.$ $\therefore p_2 = 500\ 000 + \frac{12^2 - 8.446^2}{2\times 9.81}\times 9810$

$= 536\ 300\text{ Pa}.$

Energy from 1 → 3: $\frac{V_1^2}{2g}+\frac{p_1}{g}=\frac{V_3^2}{2g}+\frac{p_3}{g}$.

$\therefore p_3 = 500\ 000+\frac{12^2-8^2}{2\times 9.81}9810 = 540\ 000$ Pa.

$p_1A_1 - p_2A_2 - R_x = \dot{m}_2V_{2x} + \dot{m}_3V_{3x} - \dot{m}_1V_{1x}$.

$\therefore R_x = 500\ 000\pi\times.03^2 - 536\ 300\pi\times.03^2 + 33.93\times 12 - 23.88\times 8.446 = \underline{103\text{ N}}$.

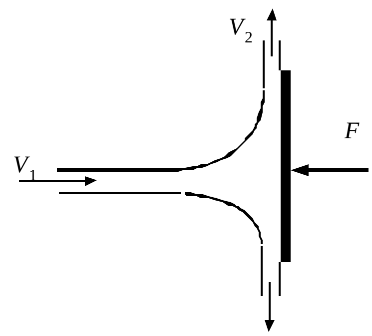
$p_3A_3 - R_y = \dot{m}_3V_{3y} + \dot{m}_2V_{2y} - \dot{m}_1V_{1y}$.

$\therefore R_y = 540\ 000\pi\times.02^2 - 10.05\times(-8) = \underline{759\text{ N}}$.

4.125 a) $\Sigma F_x = \dot{m}(V_{2x} - V_{1x})$. $\quad -F = -\dot{m}V_1$. $\quad V_1 = \frac{\dot{m}}{\rho A_1}$

$= \frac{300}{1000\pi\times.05^2} = 38.2$ m/s

$\therefore F = 300\times 38.2 = \underline{11\ 460\text{ N}}$.



b) $-F = \dot{m}_r(V_1 - V_B)(\cos\alpha - 1)$.

$\therefore F = 300\times\frac{28.2}{38.2}(38.2-10) = \underline{6250\text{ N}}$.

c) $-F = \dot{m}_r(V_1 - V_B)(\cos\alpha - 1)$.

$\therefore F = 300\times\frac{48.2}{38.2}(38.2-(-10)) = \underline{18\ 250\text{ N}}$.

4.126 a) $-F = \dot{m}(V_{2x} - V_{1x})$. $\quad 200 = 1.94\pi\left(\frac{1.25}{12}\right)^2\times V_1^2$. $\quad \therefore V_1 = \underline{55\text{ fps.}}$

b) $-F = \dot{m}_r(V_1 - V_B)(\cos\alpha - 1)$. $\quad 200 = 1.94\pi\left(\frac{1.25}{12}\right)^2(V_1 - 30)^2$. $\quad \therefore V_1 = \underline{85\text{ fps.}}$

c) $-F = \dot{m}_r(V_1 - V_B)(\cos\alpha - 1)$. $\quad 200 = 1.94\pi\left(\frac{1.25}{12}\right)^2(V_1 + 30)^2$. $\quad \therefore V_1 = \underline{25\text{ fps.}}$

4.127 a) $-F = \dot{m}(V_{2x} - V_{1x})$. $\quad -700 = 1000\pi\times.04^2V_1(V_1\times\cos 30^\circ - V_1)$. $\quad \therefore V_1 = 32.24$ m/s.

$\therefore \dot{m} = \rho A_1V_1 = 1000\pi\times.04^2\times 32.24 = \underline{162.1\text{ kg/s.}}$

b) $-F = \dot{m}_r(V_1 - V_B)(\cos\alpha - 1)$. $-700 = 1000\pi\times.04^2(V_1 - 8)^2(.866-1)$. $\therefore V_1 = 40.24$ m/s.

$\therefore \dot{m} = \rho A_1V_1 = 1000\pi\times.04^2\times 40.24 = \underline{202\text{ kg/s.}}$

c) $-F = \dot{m}_r(V_1 - V_B)(\cos\alpha - 1)$. $-700 = 1000\pi\times.04^2(V_1 + 8)^2(.866-1)$. $\therefore V_1 = 24.24$ m/s.

$\therefore \dot{m} = \rho A_1V_1 = 1000\pi\times.04^2\times 24.24 = \underline{121.8\text{ kg/s.}}$

4.128 (**D**) $\quad -F_x = \dot{m}(V_{2x} - V_{1x}) = 1000\times 0.01\times 0.2\times 50(50\cos 60^\circ - 50) = -2500$ N.

4.129 a) $-R_x = \dot{m}(V_{2x} - V_{1x}) = 1.94\pi \times \left(\frac{1}{12}\right)^2 \times 120(120\cos 60^\circ - 120).$ $\therefore R_x = \underline{305 \text{ lb}}.$

$R_y = \dot{m}(V_{2y} - V_{1y}) = 1.94\pi \left(\frac{1}{12}\right)^2 \times 120 \times (120 \times .866).$ $\therefore R_y = \underline{528 \text{ lb}}.$

b) $-R_x = \dot{m}_r(V_1 - V_B)(\cos\alpha - 1) = 1.94\pi \times \left(\frac{1}{12}\right)^2 \times 60 \times 60(.5 - 1).$ $\therefore R_x = \underline{76.2 \text{ lb}}.$

$R_y = \dot{m}_r(V_1 - V_B)\sin\alpha = 1.94\pi \left(\frac{1}{12}\right)^2 \times 60 \times (60 \times .866).$ $\therefore R_y = \underline{132 \text{ lb}}.$

c) $-R_x = \dot{m}_r(V_1 - V_B)(\cos\alpha - 1) = 1.94\pi \times \left(\frac{1}{12}\right)^2 \times 180 \times 180(.5 - 1).$ $\therefore R_x = \underline{686 \text{ lb}}.$

$R_y = \dot{m}_r(V_1 - V_B)\sin\alpha = 1.94\pi \left(\frac{1}{12}\right)^2 \times 180 \times (180 \times .866).$ $\therefore R_y = \underline{1188 \text{ lb}}.$

4.130 $V_B = R\omega = 0.5 \times 30 = 15 \text{ m/ s}.$

$-R_x = \dot{m}(V_1 - V_B)(\cos\alpha - 1) = 1000\pi \times .025^2 \times 40 \times 25(.5 - 1).$ $\therefore R_x = 982 \text{ N}.$

$\therefore \dot{W} = 10 R_x V_B = 10 \times 982 \times 15 = \underline{147\ 300 \text{ W}}.$

4.131 a) $-R_x = \dot{m}(V_{2x} - V_{1x}) = 4\pi .02^2 \times 400(-400\cos 60^\circ - 400).$ $\therefore R_x = \underline{1206 \text{ N}}.$

$R_y = \dot{m}(V_{2y} - V_{1y}) = 4\pi \times .02^2 \times 400(400\sin 60^\circ).$ $\therefore R_y = \underline{696 \text{ N}}.$

b) $-R_x = \dot{m}_r(V_1 - V_B)(\cos 120^\circ - 1) = 4\pi .02^2 \times 300^2(-.5 - 1).$ $\therefore R_x = \underline{679 \text{ N}}.$

$R_y = \dot{m}_r(V_1 - V_B)\sin\alpha = 4\pi \times .02^2 \times 300^2 \times .866.$ $\therefore R_y = \underline{392 \text{ N}}.$

c) $-R_x = \dot{m}_r(V_1 - V_B)(\cos 120^\circ - 1) = 4\pi .02^2 \times 500^2(-.5 - 1).$ $\therefore R_x = \underline{1885 \text{ N}}.$

$R_y = \dot{m}_r(V_1 - V_B)\sin\alpha = 4\pi \times .02^2 \times 500^2 \times .866.$ $\therefore R_y = \underline{1088 \text{ N}}.$

4.132 $-F_x = \dot{m}(V_1 - V_B)(\cos 120^\circ - 1) = 4\pi \times .02^2 \times (400 - 180)^2(-.5 - 1).$ $\therefore R_x = 365 \text{ N}.$

$V_B = 1.2 \times 150 = 180 \text{ m/ s}.$ $\dot{W} = 15 \times 365 \times 180 = \underline{986\ 000 \text{ W}}.$

The y-component force does no work.

4.133 (**A**) $-F_x = \dot{m}(V_{r2x} - V_{r1x}) = 1000 \times \pi \times 0.02^2 \times 60 \times (40\cos 45^\circ - 40) = 884 \text{ N}.$

Power $= F_x \times V_B = 884 \times 20 = 17700 \text{ W}.$

4.134 a) Refer to Fig. 4.16: $\left.\begin{array}{l} 750\sin\beta_1 = V_{r1}\sin 45^\circ \\ 750\cos\beta_1 - 300 = V_{r1}\cos 45^\circ \end{array}\right\}$ $\therefore$ $V_{r1} = 507 \text{ fps.}$ $= V_{r2}$

Note: $V_{2x} - V_{1x} = -V_{r2}\cos\alpha_2 + V_B - V_{r1}\cos\alpha_1 - V_B = -V_{r1}(\cos\alpha_2 + \cos\alpha_1).$

$\therefore R_x = \dot{m}V_{r1}(\cos\alpha_2 + \cos\alpha_1) = .015\pi \left(\frac{.5}{12}\right)^2 \times 750 \times 507(\cos 30^\circ + \cos 45^\circ) = 48.9 \text{ lb}.$

$$\therefore \dot{W} = 15R_xV_B = 15\times 48.9\times 300 = 220{,}000\,\frac{\text{ft - lb}}{\text{sec}} \quad \text{or} \quad \underline{400\text{ Hp.}}$$

b) $$\left.\begin{aligned} 750\sin\beta_1 &= V_{r1}\sin 60^\circ \\ 750\cos\beta_1 - 300 &= V_{r1}\cos 60^\circ \end{aligned}\right\} \quad V_{r1} = 554 \text{ fps} = V_{r2}.$$

$$\therefore R_x = \dot{m}V_{r1}(\cos\alpha_2 + \cos\alpha_1) = .015\pi\times\left(\frac{.5}{12}\right)^2\times 750\times 554(\cos 30^\circ + \cos 60^\circ) = 46.4 \text{ lb.}$$

$$\therefore \dot{W} = 15R_xV_B = 15\times 46.4\times 300 = 209{,}000\,\frac{\text{ft - lb}}{\text{sec}} \quad \text{or} \quad \underline{380\text{ Hp.}}$$

c) $$\left.\begin{aligned} 750\sin\beta_1 &= V_{r1}\sin 90^\circ \\ 750\cos\beta_1 - 300 &= V_{r1}\cos 90^\circ \end{aligned}\right\} \quad V_{r1} = 687 \text{ fps} = V_{r2}.$$

$$\therefore R_x = \dot{m}V_{r1}(\cos\alpha_2 + \cos\alpha_1) = .015\pi\times\left(\frac{.5}{12}\right)^2\times 750\times 687(\cos 30^\circ + 0) = 36.5 \text{ lb.}$$

$$\therefore \dot{W} = 15R_xV_B = 15\times 36.5\times 300 = 164{,}300\,\frac{\text{ft - lb}}{\text{sec}} \quad \text{or} \quad \underline{299\text{ Hp.}}$$

4.135 a) Refer to Fig. 4.16: $$\left.\begin{aligned} 100\sin 30^\circ &= V_{r1}\sin\alpha_1 \\ 100\cos 30^\circ - 20 &= V_{r1}\cos\alpha_1 \end{aligned}\right\} \quad \therefore \alpha_1 = \underline{36.9^\circ},\quad V_{r1} = 83.3 \text{ m/s.}$$

$$\left.\begin{aligned} V_2\sin 60^\circ &= 83.3\sin\alpha_2 \\ V_2\cos 60^\circ &= 83.3\cos\alpha_2 - 20 \end{aligned}\right\} \quad V_2 = 71.5,\quad \alpha_2 = \underline{48^\circ}.$$

$$-R_x = \dot{m}(V_{2x} - V_{1x}) = 1000\pi\times .015^2\times 100(-71.5\cos 60^\circ - 100\cos 30^\circ). \quad \therefore R_x = 8650 \text{ N.}$$

$$\therefore \dot{W} = 12V_BR_x = 12\times 20\times 8650 = \underline{2.08\times 10^6 \text{ W}}.$$

b) $$\left.\begin{aligned} 100\sin 30^\circ &= V_{r1}\sin\alpha_1 \\ 100\cos 30^\circ - 40 &= V_{r1}\cos\alpha_1 \end{aligned}\right\} \quad \therefore \alpha_1 = \underline{47^\circ},\quad V_{r1} = V_{r2} = 68.35 \text{ m/s.}$$

$$\left.\begin{aligned} V_2\sin 60^\circ &= 68.35\sin\alpha_2 \\ V_2\cos 60^\circ &= 68.35\cos\alpha_2 - 40 \end{aligned}\right\} \quad V_2 = 38.9 \text{ m/s},\quad \alpha_2 = \underline{29.5^\circ}.$$

$$-R_x = \dot{m}(V_{2x} - V_{1x}) = 1000\pi\times .015^2\times 100(-38.9\cos 60^\circ - 100\cos 30^\circ). \quad \therefore R_x = 7500 \text{ N.}$$

$$\therefore \dot{W} = 12V_BR_x = 12\times 40\times 7500 = \underline{3.60\times 10^6 \text{ W}}.$$

c) $$\left.\begin{aligned} 100\sin 30^\circ &= V_{r1}\sin\alpha_1 \\ 100\cos 30^\circ - 50 &= V_{r1}\cos\alpha_1 \end{aligned}\right\} \quad \therefore \alpha_1 = \underline{53.8^\circ},\quad V_{r1} = V_{r2} = 61.96 \text{ m/s.}$$

$$\left.\begin{aligned} V_2\sin 60^\circ &= 61.76\sin\alpha_2 \\ V_2\cos 60^\circ &= 61.96\cos\alpha_2 - 50 \end{aligned}\right\} \quad V_2 = 19.32 \text{ m/s},\quad \alpha_2 = \underline{15.66^\circ}.$$

$$-R_x = \dot{m}(V_{2x} - V_{1x}) = 1000\pi \times .015^2 \times 100(-19.32\cos 60^\circ - 100\cos 30^\circ). \quad \therefore R_x = 6800 \text{ N}.$$

$$\therefore \dot{W} = 12 R_x V_B = 12 \times 6800 \times 50 = \underline{4.08 \times 10^6 \text{ W}}.$$

4.136 a) Refer to Fig. 4.16: $\left.\begin{array}{l} 50\sin 30^\circ = V_{r1}\sin\alpha_1 \\ 50\cos 30^\circ - V_B = V_{r1}\cos\alpha_1 \end{array}\right\} \quad \therefore V_{r1}^2 = 2500 - 86.6V_B + V_B^2$

$$\left.\begin{array}{l} 30\sin 60^\circ = V_{r2}\sin\alpha_2 \\ 30\cos 60^\circ - V_{r2}\cos\alpha_2 = V_B \end{array}\right\} \quad \therefore V_{r2}^2 = V_{r1}^2 = 900 + 30V_B + V_B^2.$$

Combine the above: $V_B = 13.72$ m / s. Then, $\alpha_1 = \underline{59.4^\circ}$, $\alpha_2 = \underline{42.1^\circ}$.

$$-R_x = \dot{m}(V_{2x} - V_{1x}) = 1000\pi \times .01^2 \times 50(-30\cos 60^\circ - 50\cos 30^\circ). \quad \therefore R_x = 916 \text{ N}.$$

$$\therefore \dot{W} = 15 V_B R_x = 15 \times 13.72 \times 916 = \underline{188\ 500 \text{ W}}.$$

b) $\left.\begin{array}{l} 50\sin 30^\circ = V_{r1}\sin\alpha_1 \\ 50\cos 30^\circ - V_B = V_{r1}\cos\alpha_1 \end{array}\right\} \quad \therefore V_{r1}^2 = 2500 - 86.6V_B + V_B^2 \quad \therefore V_B = 14.94$ m / s.

$$\left.\begin{array}{l} 30\sin 70^\circ = V_{r2}\sin\alpha_2 \\ 30\cos 70^\circ - V_{r2}\cos\alpha_2 = V_B \end{array}\right\} \quad \therefore V_{r2}^2 = 900 + 20.52V_B + V_B^2. \ \alpha_1 = \underline{41.4^\circ}, \alpha_2 = \underline{48.2^\circ}$$

$$-R_x = \dot{m}(V_{2x} - V_{1x}) = 1000\pi \times .01^2 \times 50(-30\cos 70^\circ - 50\cos 30^\circ). \quad \therefore R_x = 841 \text{ N}.$$

$$\therefore \dot{W} = 15 V_B R_x = 15 \times 14.94 \times 841 = \underline{188\ 500 \text{ W}}.$$

c) $\left.\begin{array}{l} 50\sin 30^\circ = V_{r1}\sin\alpha_1 \\ 50\cos 30^\circ - V_B = V_{r1}\cos\alpha_1 \end{array}\right\} \therefore V_{r1}^2 = 2500 - 86.6V_B + V_B^2 \quad \therefore V_B = 16.49$ m / s

$$\left.\begin{array}{l} 30\sin 80^\circ = V_{r2}\sin\alpha_2 \\ 30\cos 80^\circ - V_{r2}\cos\alpha_2 = V_B \end{array}\right\} \quad \therefore V_{r2}^2 = 900 + 10.42V_B + V_B^2. \ \alpha_1 = \underline{43^\circ}, \ \alpha_2 = \underline{53.7^\circ}$$

$$-R_x = \dot{m}(V_{2x} - V_{1x}) = 1000\pi \times .01^2 \times 50(-30\cos 80^\circ - 50\cos 30^\circ). \quad \therefore R_x = 762 \text{ N}.$$

$$\therefore \dot{W} = 15 V_B R_x = 15 \times 16.49 \times 762 = \underline{188\ 500 \text{ W}}.$$

4.137 To find F, sum forces normal to the plate: $\Sigma F_n = \dot{m}\left[\cancel{(V_{out})_n} - V_{1n} \right]$.

a) $\therefore F = 1000 \times .02 \times .4 \times 40\left[-(-40\sin 60^\circ)\right] = \underline{11\ 080 \text{ N}}$. (We have neglected friction)

$$\Sigma F_t = 0 = \dot{m}_2 V_2 + \dot{m}_3(-V_3) - \dot{m}_1 \times 40\sin 30^\circ. \quad \text{Bernoulli: } V_1 = V_2 = V_3.$$

$$\left.\begin{array}{l} \therefore \quad 0 = \dot{m}_2 - \dot{m}_3 - .5\dot{m}_1 \\ \text{Continuity: } \dot{m}_1 = \dot{m}_2 + \dot{m}_3 \end{array}\right\} \quad \begin{array}{l} \therefore \dot{m}_2 = .75\dot{m}_1 = .75 \times 320 = \underline{240 \text{ kg / s.}} \\ \dot{m}_3 = \underline{80 \text{ kg / s.}} \end{array}$$

b) $\therefore F = -1.94 \times \frac{1}{12} \times \frac{20}{12} \times 120(-120\sin 60^\circ) = \underline{3360 \text{ lb}}$. (We have neglected friction)

$$\Sigma F_t = 0 = \dot{m}_2 V_2 + \dot{m}_3(-V_3) - \dot{m}_1 \times 120\sin 30^\circ. \quad \text{Bernoulli: } V_1 = V_2 = V_3.$$

$$\therefore \quad 0 = \dot m_2 - \dot m_3 - 0.5\dot m_1 \quad \text{Continuity: } \dot m_1 = \dot m_2 + \dot m_3 \Bigg\} \quad \therefore \dot m_2 = .75\dot m_1 = .75\times 1.94\times \frac{20}{144}\times 120$$

$$= \underline{22.6 \text{ slug/sec.}} \text{ and } \dot m_3 = \underline{9.7 \text{ slug/sec.}}$$

4.138 $F = \dot m_r (V_{1r})_n = 1000\times.02\times.4\times(40+20)^2 \sin 60^\circ = 24\,940$ N.

$F_x = 24\,940 \cos 30^\circ = 21\,600$ N. $\quad \therefore \dot W = 21\,600\times 20 = \underline{432\,000 \text{ W}}$.

4.139 $F = \dot m_r (V_{1r})_n = 1000\times.02\times.4(40 - V_B)^2 \sin 60^\circ$. $\quad F_x = 8(40 - V_B^2)\sin^2 60^\circ$.

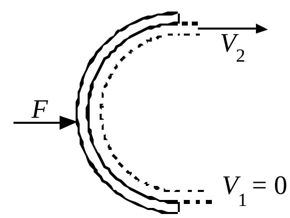
$\dot W = V_B F_x = 8V_B(40 - V_B)^2\times.75 = 6(1600V_B - 80V_B^2 + V_B^3)$.

$\dfrac{d\dot W}{dV_B} = 6(1600 - 160V_B + 3V_B^2) = 0. \quad \therefore V_B = \underline{13.33 \text{ m / s.}}$

4.140 (**A**) Let the vehicle move to the right. The scoop then diverts the water to the right. Then

$$F = \dot m(V_{2x} - V_{1x}) = 1000\times 0.05\times 2\times 60\times[60-(-60)] = 720000 \text{ N.}$$

4.141 $F = \dot m_r(\cancel{V_1} - V_B)(\cos\alpha - 1) = 1000\times.1\times.6V_B(-V_B)(-2) = 120V_B^2$.



At $t = 0$: $\quad F = 120\times\left(\dfrac{120\times 1000}{3600}\right)^2 = 133\,300$ N.

$a_o = \dfrac{133\,300}{100\,000} = \underline{1.33 \text{ m/s}^2}$

$\dfrac{-F}{m} = \dfrac{dV_B}{dt} = \dfrac{-120V_B^2}{100\,000}. \quad \therefore \displaystyle\int_{33.33}^{16.67} -\dfrac{dV_B}{V_B^2} = .0012\int_0^t dt.$

$\therefore \left[\dfrac{1}{16.67} - \dfrac{1}{33.33}\right] = .0012\ \text{t}. \quad \therefore t = \underline{26.6 \text{ sec}}.$

4.142 $F = \dot m_r(V_1 - V_B)(\cos\alpha - 1) = 90\times.8\times 2.5\times 13.89\times(-13.89)(-1) = 34700$ N.

$\left(V_B = \dfrac{50\times 1000}{3600} = 13.89 \text{ m/s}\right) \quad \therefore \dot W = 34700\times 13.89 = 482\,000 \text{ W} \quad \text{or} \quad \underline{647 \text{ Hp.}}$

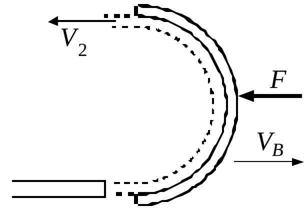
4.143 See the figure in Problem 4.141.

$F = \dot m_r(V_1 - V_B)(\cos\alpha - 1) = 1000\times.06\times.2\times V_B(-V_B)(-2) = 24\ \text{V}_\text{B}^2$.

$-F = mV_B\dfrac{dV_B}{dx}. \quad \therefore -24V_B^2 = 5000V_B\dfrac{dV_B}{dx}.$

$-\displaystyle\int_0^x \dfrac{24\ dx}{5000} = \int_{250}^{27.78}\dfrac{dV_B}{V_B}. \quad -\dfrac{24}{5000}x = \ell n 27.78 - \ell n 250. \quad \therefore x = \underline{458 \text{ m}}.$

4.144 $-F = \dot{m}_r(V_1 - V_B)(\cos a - 1) = 1.94p \times \left(\frac{1.25}{12}\right)^2 (V_1 - V_B)^2(-2).$



$$\therefore F = 0.1323(V_1 - V_B)^2 = 20\frac{dV_B}{dt}.$$

At $t = 0, \quad V_B = 0.$ Then $20\frac{dV_B}{dt} = 0.1323V_1^2.$

With $\frac{dV_B}{dt} = 6, \; V_1 = \underline{30.1 \text{ fps}}.$

For $t > 0, \quad \int_0^{V_B} \frac{dV_B}{(30.1 - V_B)^2} = 0.006615\int_0^2 dt. \quad 0.01323 = \frac{1}{30.1 - V_B} - \frac{1}{30.1}. \quad \therefore V_B = \underline{8.57 \text{ fps}}.$

4.145 For this steady-state flow, we fix the boat and move the upstream air. This provides us with the steady-state flow of Fig. 4.17. This is the same as observing the flow while standing on the boat.

$\dot{W} = FV_1. \quad 20\,000 = F\frac{50 \times 1000}{3600}. \quad \therefore F = \underline{1440 \text{ N}}. \quad (V_1 = 13.89 \text{ m/s})$

$F = \dot{m}(V_2 - V_1). \quad 1440 = 1.23p \times 1^2 \frac{V_2 + 13.89}{2}(V_2 - 13.89). \quad \therefore V_2 = 30.6 \text{ m/s}.$

$\therefore Q = A_3V_3 = p \times 1^2 \frac{30.6 + 13.89}{2} = \underline{69.9 \text{ m}^3\text{/s}}.$

$h_p = \frac{V_1}{V_3} = \frac{13.89}{22.24} = 0.625$ or $\underline{62.5\%}.$

4.146 Fix the reference frame to the aircraft so that $V_1 = \frac{200 \times 1000}{3600} = 55.56 \text{ m/s}.$

$V_2 = \frac{320 \times 1000}{3600} = 88.89 \text{ m/s}. \quad \therefore \dot{m} = 1.2 \times p \times 1.1^2 \frac{55.56 + 88.89}{2} = 329.5 \text{ kg/s}.$

$F = 329.5(88.89 - 55.56) = 10\,980 \text{ N}.$

$= \Delta p p \times 1.1^2. \quad \therefore \Delta p = \underline{2890 \text{ Pa}}.$

$\dot{W} = F \times V_1 = 10\,980 \times 55.56 = 610\,000 \text{ W}$ or $\underline{818 \text{ Hp}}.$

4.147 Fix the reference frame to the boat so that $V_1 = 20 \times \frac{88}{60} = 29.33 \text{ fps}.$

$V_2 = 40 \times \frac{88}{60} = 58.67 \text{ fps}. \quad \therefore F = \dot{m}(V_2 - V_1) = 1.94p \times \left(\frac{10}{12}\right)^2 \frac{29.33 + 58.67}{2}(58.67 - 29.33)$

$= 5460 \text{ lb}.$

$\dot{W} = F \times V_1 = 5460 \times 29.33 = 160{,}000 \frac{\text{ft-lb}}{\text{sec}}$ or $\underline{291 \text{ Hp}}.$

$$\dot{m} = 1.94 \times p \times \left(\frac{10}{12}\right)^2 \frac{29.33 + 58.67}{2} = \underline{186.2 \text{ slug/ sec.}}$$

4.148 Fix the reference frame to the boat: $V_1 = 10$ m / s, $V_2 = 20$ m / s.

$\therefore$Thrust = $\dot{m}(V_2 - V_1) = 1000 \times 0.2(20 - 10) = \underline{2000 \text{ N}}$.

$\dot{W} = F \times V_1 = 2000 \times 10 = 20\ 000$ W or $\underline{26.8 \text{ Hp}}$.

4.149 $0.2 = \overline{V}_1 A_1 = \overline{V}_1 \times .2 \times 1.0$. $\therefore \overline{V}_1 = 1$ m / s. $\therefore V_{1\,\max} = 2$ m / s. $\therefore V_1(y) = 20(0.1 - y)$.

$$\text{flux in} = 2\int_0^{.1} rV^2 dy = 2\int_0^{.1} 1000 \times 20^2 (.1 - y)^2 dy = 800\ 000 \frac{.1^3}{3} = 267 \text{ N}.$$

The slope at section 1 is -20. $\therefore V_2(y) = -20y + A$.

Continuity: $A_1\overline{V}_1 = A_2\overline{V}_2$. $\therefore \overline{V}_2 = 2\overline{V}_1 = 2$ m / s. $\left.\begin{matrix} V_2(0) = A \\ V_2(.05) = A - 1 \end{matrix}\right\} \therefore \overline{V}_2 = A - 1/\ 2.$

$2 = A - 1/\ 2$. $\therefore A = 2.5$. $\therefore V_2(y) = 2.5 - 20y$.

$$\text{flux out} = 2\int_0^{.05} 1000(2.5 - 20y)^2 dy = 800\ 000\left[\frac{(y - .125)^3}{3}\right]_0^{.05} = \frac{800\ 000}{3}[0.00153]$$

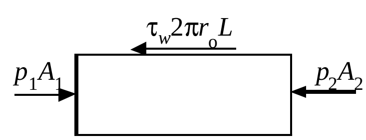
$= 408.3$ N. $\therefore$change = 408 – 267 = $\underline{141 \text{ N.}}$

4.150 a) $$b = \frac{\int V^2 dA}{\overline{V}^2 A} = \frac{2\int_0^{.1} 20^2(.1 - y)^2 dy}{1^2 \times .2 \times 1.0} = 4000\frac{.1^3}{3} = \underline{\frac{4}{3}}.$$

b) See Problem 4.149: $V_2(y) = 20(0.125 - y)$, $.05 \geq y \geq 0$. $\overline{V}_2 = 2$ m / s.

$$b = \frac{\int V^2 dA}{\overline{V}^2 A} = \frac{2\int_0^{.05} 20^2 (y - .125)^2 dy}{2^2 \times .1 \times 1.0} = 2000\frac{(y - .125)^3}{3}\Big|_0^{.05} = \underline{1.021}.$$

4.151 From the c.v. shown: $(p_1 - p_2)pr_0^2 = t_w 2pr_o L$.



$$\therefore t_w = \frac{\Delta p\, r_o}{2L} = m\frac{du}{dr}\bigg|_w. \qquad \therefore \frac{du}{dr}\bigg|_w = \frac{0.03 \times 144 \times .75/\ 12}{2 \times 30 \times 2.36 \times 10^{-5}}$$

$$= \underline{191 \frac{\text{ft / sec}}{\text{ft}}}.$$

4.152 Write the equation of the parabola: $V(r) = V_{\max}\left(1 - \frac{r^2}{r_0^2}\right)$.

Continuity: $p\times.006^2\times 8=\int_0^{.006} V_{\max}\left(1-\frac{r^2}{.006^2}\right)2prdr. \quad \therefore V_{\max}=16\text{ m / s}.$

Momentum: $p_1A_1-p_2A_2-F_{\text{Drag}}=\int rV^2dA-\dot{m}V_1.$

$$40\,000p\times.006^2-F_{\text{Drag}}=\int_0^{.006}1000\times 16^2\left(1-\frac{r^2}{.006^2}\right)^2 2prdr-1000\times p\times.006^2\times 8\times 8$$

$$4.524-F_{\text{Drag}}=9.651-7.238. \qquad \therefore F_{\text{Drag}}=\underline{2.11\text{ N}}.$$

4.153 $\dot{m}_{top}=rA_1V_1-r\int V_2(y)dA=1.23\left[2\times 10\times 32-\int_0^2(28+y^2)10dy\right]=65.6\text{ kg / s}.$

$$-\frac{F}{2}=\int rV^2dA+\dot{m}_{top}V_1-\dot{m}_1V_1=1.23\int_0^2(28+y^2)^2\,10dy+65.6\times 32-1.23\times 20\times 32^2.$$

$$\therefore F=\underline{3780\text{ N}}.$$

4.154 a) $\dot{m}_{top}=\dot{m}_1-\dot{m}_2=rA_1V_1-\int ru(y)dA=1.23\left[.1\times 2\times 8-\int_0^{.1}(20y-100y^2)8\times 2dy\right]$

$=0.656\text{ kg/ s}.$ (Note: $y=0.1$ for $u(y)=8$).

Momentum: $-F_{\text{Drag}}=r\int_0^{.1}64(20y-100y^2)^2\,2dy+.656\times 8-r\times.1\times 2\times 8^2$

$=1.23\times 6.83+5.25-1.23\times 12.8. \qquad \therefore F_{\text{Drag}}=\underline{2.1\text{ N}}$

b) To find h: $\quad 8h=\int_0^{.1}8(20y-100y^2)dy.$

$$\therefore h=\frac{20\times.1^2}{2}-\frac{100\times.001}{3}=0.0667\text{ m}.$$

Momentum: $-F_{\text{Drag}}=1.23\int_0^{.1}64(20y-100y^2)^2\,2dy-1.23\times.0667\times 2\times 8^2.$

$=1.23\times 6.83-10.50. \qquad \therefore F_{\text{Drag}}=\underline{2.1\text{ N}}.$

4.155 a) Energy: $\frac{V_1^2}{2g}+z_1=\frac{V_2^2}{2g}+z_2+h_L.$ See Problem 4.118(a).

$\frac{8^2}{2\times 9.81}+0.6=\frac{1.912^2}{2\times 9.81}+2.51+h_L. \qquad \therefore h_L=1.166\text{ m}.$

$\therefore$ losses $=gA_1V_1h_L=9810\times(.6\times 1)\times 8\times 1.166=\underline{54\,900\text{ W/ m}}$ of width.

b) See Problem 4.120: $\frac{V_1^2}{2g}+z_1=\frac{V_2^2}{2g}+z_2+h_L.$

$$\frac{7.19^2}{2\times 9.81} + .417 = \frac{1.58^2}{2\times 9.81} + 1.9 + h_L. \qquad \therefore h_L = 1.025 \text{ m}.$$

$\therefore$ losses $= gA_1V_1h_L = 9810 \times .417 \times 3 \times 7.19 \times 1.025 = \underline{90\ 300\text{ W}}$

c) See Problem 4.121: $\frac{5.17^2}{2\times 9.81} + 1.16 = \frac{3^2}{2\times 9.81} + 2 + h_L. \quad \therefore h_L = 0.0636$ m.

$\therefore$ losses $= gA_1V_1h_L = 9810 \times 1.16 \times 5.17 \times 0.0636 = \underline{3740\text{ W / m}}$ of width.

4.156 See Problem 4.122: $V_1 = 20$ m / s, $V_2 = 5$ m / s, $p_1 = 60$ kPa, $p_2 = 135$ kPa.

Then, $\frac{V_1^2}{2g} + \frac{p_1}{g} = \frac{V_2^2}{2g} + \frac{p_2}{g} + h_L. \quad \frac{20^2}{2\times 9.81} + \frac{60\ 000}{9810} = \frac{5^2}{2\times 9.81} + \frac{135\ 000}{9810} + h_L.$

$$\therefore h_L = 11.47 \text{ m} = K\frac{V_1^2}{2g} = K\frac{20^2}{2\times 9.81}. \qquad \therefore K = \underline{0.562}.$$

4.157 Continuity: $V_1 D^2 = Vd^2. \qquad \therefore V_1 = \frac{d^2}{D^2}V.$

Energy: $\frac{V_1^2}{2g} + H(t) = \frac{V^2}{2g}. \qquad \therefore V = \sqrt{2gH(t)}.$

Momentum: $\Sigma F_x - (F_I)_x = \frac{d}{dt}\int_{c.v.} rV_x\, d\Psi + \dot{m}(V_{2x} - V_{1x}). \quad \left(\frac{d^2\bar{s}}{dt^2}\right)_x = a_x.$

$$\therefore -a_x m(t) = r\frac{pd^2}{4}V(V). \quad m(t) = m_o - r\int_0^t \frac{pd^2}{4}V(t)dt.$$

But, $V_1 = -\frac{dH}{dt}. \quad \therefore -\frac{dH}{dt} = \frac{d^2}{D^2}\sqrt{2gH}. \quad \therefore -\frac{dH}{H^{1/2}} = \frac{d^2}{D^2}\sqrt{2g}dt. \quad \therefore H^{1/2} = \frac{\sqrt{2g}d^2}{2D^2}t + \sqrt{H_o}.$

$$\therefore a_x = \frac{rpd^2}{4}2g\left(\frac{\sqrt{2g}d^2}{2D^2}t + \sqrt{H_o}\right)^2 \left[r\frac{pd^2}{4}\int_0^t \sqrt{2g}\left(\frac{\sqrt{2g}d^2}{2D^2}t + \sqrt{H_o}\right)dt - m_o\right]$$

4.158 This is a very difficult design problem. There is an optimum initial mass of water for a maximum height attained by the rocket. It will take a team of students many hours to work this problem. It involves continuity, energy, and momentum.

4.159 $V_e = \frac{\dot{m}}{rA_e} = \frac{4}{1000 \times 4 \times p \times .004^2} = 19.89$ m / s. Velocity in arm $= V$.

$$\vec{M}_I = \int_{c.v.} \vec{r} \times (2\Omega \times \vec{V}) r\, d\Psi = 4\int_0^{.3} r\hat{i} \times (-2\Omega\hat{k} \times V\hat{i}) rAdr$$

$$= -8\rho A V\Omega \hat{k}\int_0^{.3} r dr = -0.36\rho A V\Omega \hat{k}.$$

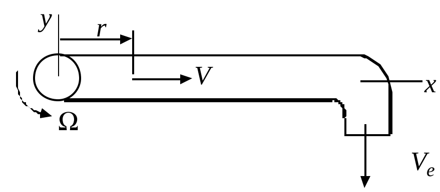
$\Sigma \vec{M} = 0$ and $\frac{d}{dt}\int_{c.v.}(\vec{r}\times\vec{V})\rho d\forall = 0.$ $\int_{c.s.}\vec{r}\times\vec{V}(\vec{V}\cdot\hat{n})\rho dA = .3\hat{i}\times(.707V_e\hat{j} + .707V_e\hat{k})V_e\rho A_e.$

The z-component of $\int_{c.s.}\vec{r}\times\vec{V}(\vec{V}\cdot\hat{n})\rho dA = .3\times .707V_e^2 A_e\rho.$

Finally, $-(M_I)_z = 0.36\rho AV\Omega = 4\times .3\times .707V_e^2 A_e \rho.$ Using $AV = A_eV_e$,

$0.36\Omega = 4\times .3\times .707\times 19.89.$ $\therefore \Omega = \underline{46.9 \text{ rad/s}}.$

4.160 A moment $\vec{M}$ resists the motion thereby producing power. One of the arms is shown.



$$\vec{M}_I = \int_0^{.25} 4r\hat{i}\times(-2\Omega\hat{k}\times V\hat{i})\rho A dr = -8\rho AV\Omega\hat{k}\int_0^{10/12} r dr = -2.778\rho AV\Omega\hat{k}.$$

$$\Sigma\vec{M} = M\hat{k},\ \frac{d}{dt}\int_{c.v.}(\vec{r}\times\vec{V})d\forall = 0, \text{ and } \int_{c.s.}\vec{r}\times\vec{V}(\vec{V}\cdot\hat{n})\rho dA = \frac{10}{12}\times V_e^2\rho A_e\times 4\hat{k}.$$

Thus, $M + 2.778\times 1.94\pi\left(\frac{.75}{12}\right)^2\times\frac{200}{9}\times 30 = 200^2\times\frac{10}{12}\times 1.94\pi\left(\frac{1/4}{12}\right)^2\times 4.$

$\therefore M = 309$ ft-lb.

$\dot{W} = M\Omega = 309\times 30 = \underline{9270 \text{ ft-lb/sec}}.$

4.161 $\dot{m} = 10 = \rho AV = 1000\pi\times .01^2 V_0.$ $\therefore V_0 = 31.8$ m/s.

Continuity: $V_0\pi\times .01^2 = V\pi\times .01^2 + V_e\times .006(r - .05).$

$V_0\pi\times .01^2 = V_e\times .006\times .15.$ $\therefore V_e = 11.1$ m/s.

$\therefore V = V_0 - 19.1(r - .05)V_e = 42.4 - 212r.$

$$\vec{M}_I = \int_0^{.05} 2r\hat{i}\times(+2\Omega\hat{k}\times V_0\hat{i})\rho A dr + \int_{.05}^{.2} 2r\hat{i}\times[+2\Omega\hat{k}\times(42.4 - 212r)\hat{i}]\rho A dr$$

$$= 4\Omega V_0\rho A\hat{k}\int_0^{.05} r dr + 4\Omega\rho A\hat{k}\int_{.05}^{.2}(42.4r - 212r^2)dr$$

$$= 4\Omega\times 31.8\times 1000\pi\times .01^2\times\frac{.05^2}{2}\hat{k} + 4\Omega\times 1000\pi\times .01^2$$

$$\left[\frac{42.4}{2}(.2^2 - .05^2) - \frac{212}{3}(.2^3 - .05^3)\right]\hat{k}$$

$$= (0.05\Omega + 0.3\Omega)\hat{k} = 0.35\Omega\hat{k}.$$

$$\int_{.05}^{.2} r\hat{i} \times (-V_e\hat{j})V_e r \times .006 dr = -11.1^2 \times 1000 \times .006 \int_{.05}^{.2} r dr\, \hat{k} = -13.86\hat{k}.$$

$\therefore -0.35\Omega = -13.86.$ $\quad\therefore \Omega = \underline{39.6 \text{ rad / s}}.$

4.162 $1000 = M\Omega. \quad \therefore M = \frac{1000}{500} = 2 \text{ N} \cdot \text{m}.$

$$\vec{M}_I = \int r\hat{i}_r \times (-2\Omega\hat{k} \times V(r)\hat{i}_r) r 2\pi r \times .02 dr$$

$$= 0.08\pi\Omega \int_0^R r^2 V(r) dr \hat{k}.$$

Continuity: $V(r)2\pi r \times .02 = V_r \cos 30^\circ 2\pi R \times .02. \quad \therefore V(r) = 0.866 R V_r / r.$

$$\int_{c.s.} \vec{r} \times \vec{V}(\vec{V} \cdot \hat{n}) r dA = -R(R\Omega + V_r \sin 30^\circ)V_r \cos 30^\circ\, r 2\pi R \times .02\hat{k} = -.00301 V_r (35 + .5V_r)\hat{k}.$$

$$\therefore -2 - 16.32 V_r \int_0^{.15} r\, dr = -.00301 V_r (35 + .5V_r). \quad \therefore V_r^2 - 52.1 V_r - 1333 = 0.$$

$$\therefore V_r = \frac{1}{2}(52.1 \pm \sqrt{52.1^2 + 4 \times 1333}) = 70.9 \text{ m / s}.$$

The flow rate is $Q = A_e V_r \cos 30^\circ = 2\pi \times .15 \times .02 \times 70.9 \times .866 = \underline{1.16 \text{ m}^3 / \text{s}}.$

4.163 See Problem 4.159. $V_e = 19.89$ m / s. $\quad V = \frac{.008^2}{.02^2} \times 19.89 = 3.18$ m / s.

$$\vec{M}_I = 4\int_0^{.3} r\hat{i} \times \left[(-2\Omega\hat{k} \times V\hat{i}) + \left(-\frac{d\Omega}{dt}\hat{k}\right) \times r\hat{i}\right] \rho A dr. \quad A = \pi \times .01^2,\ A_e = \pi \times .004^2.$$

$$= -8\rho A V\Omega\hat{k}\int_0^{.3} r dr - 4\rho A \frac{d\Omega}{dt}\hat{k}\int_0^{.3} r^2 dr = -360 A V \Omega\hat{k} - 36 A \frac{d\Omega}{dt}\hat{k}.$$

$$\int_{c.s.} (\vec{r} \times \vec{V})_z (\vec{V} \cdot \hat{n}) \rho dA = 212 V_e^2 A_e \hat{k}.$$

Thus, $360 A V \Omega + 36 A \frac{d\Omega}{dt} = 212 V_e^2 A_e$ or $\frac{d\Omega}{dt} + 31.8\Omega = 373.$

The solution is $\Omega = Ce^{-31.8t} + 11.73.$

The initial condition is $\Omega(0) = 0.$ $\quad \therefore C = -11.73.$

Finally, $\quad \Omega = \underline{11.73(1 - e^{-31.8t}) \text{ rad / s}}.$

4.164 This design problem would be good for a team of students to do as a project. How large a horsepower blower could be handled by an average person?